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Comparison of Paths to Calculus Success

1.0 Introduction and Background

Successful completion of a calculus sequence is essential to the education of any aspiring engineer. Many students have difficulty completing this task. Difficulty in succeeding in calculus is one of the primary reasons students transfer out of engineering. Even with the availability of targeted calculus topic review sessions and tutoring services, success in math has been a stumbling block for engineering students at the authors' institution and has, indeed, been identified as a critical component to engineering retention. College policy enforces the need to be successful in early math courses by: (1) requiring entering students to take a math placement exam to determine appropriate initial math course placement; (2) requiring a grade of C or better as a pre-requisite to move to the next math course in sequence (this requirement exists for all math courses prior to Differential Equations); and (3) transferring out students who earn a D or F two times in any math course up to and including Calculus 1. So, for example, if an entering freshman places into College Trigonometry and earns a D in it during his first semester, repeats that course during his second semester and earns a B, then takes Calculus 1 and earns a D in his first attempt, that student is transferred out of engineering and sent to "General Studies" where he is advised on other majors that might be better suited to his interests and abilities. The goal of these policies is to identify those students who are not succeeding in engineering in time to re-direct their focus to completing a different major so they can still graduate in as close to four years as possible. If a student, however, is determined to complete an engineering major, he may transfer back to engineering once he has successfully completed Calculus 1 with a C or better. Clearly, success in mathematics is critical to engineering retention; especially since students who struggle with mathematics may choose to leave on their own or be transferred out by policy.

The issue is not unique to one institution. It is no surprise that success in math courses is a critical component to success in most engineering programs. In fact, historically, calculus has served as a filter in many engineering schools[1]. Engineering schools and math departments in a variety of universities have worked together to address this problem. Approaches include: introducing additional cooperative learning and problem-solving opportunities for engineering students taking calculus[1,2]; creating learning communities based on math placement so students can help and encourage each other[3]; removing pre-requisites to permit students to take Calculus 1 in their second semester and still progress on schedule in their engineering curricula[4]; redefining how engineering math is taught and creating a hands-on, application-oriented approach addressing only topics relevant to the core engineering courses[5]; and instituting an "early warning" system with optional "intensive pre-calculus" mid-semester math tracks for struggling students[6,7,8]. Many of these approaches have been successful in increasing student success in calculus and in increasing engineering retention at the specific university in which the method was implemented. Each university environment has a unique set of characteristics, policies and culture. What works effectively in one university, may not be easily accomplished or as effective in another.

The mathematics department at West Virginia University, a large land grant university in the mid-Atlantic region, implemented an extended, 2-semester, Calculus 1 sequence to facilitate success for those students who arrive at the institution only marginally prepared to enter a conventional semester-long Calculus 1 course. The extended Calculus 1 course teaches the conventional Calculus 1 content, but provides "just-in-time" review of related algebraic and trigonometric concepts as they are needed throughout the course. Calculus concepts are spaced further apart to make room for the pre-calculus concept review to be interspersed throughout the course, as appropriate. Teaching standard calculus in this slower format raised questions and concerns among faculty in both the mathematics department and the engineering college regarding the impact of the slower paced calculus course:

(1) Does the slower paced extended calculus course prepare students to face the rigors of a faster paced Calculus 2 course successfully? Or, does the slower paced extended calculus course give a student false hope or a false sense of security in completing the calculus sequence, since the other courses in the calculus sequence are taught at a much faster pace? There is concern that some students who may be able to pass the slower paced calculus course, but who may not be able to pass the faster paced math courses will simply delay transferring out of engineering until, nearly, their junior year, thereby making it difficult to complete a different major within a typical 4 or 5 year college experience.

(2) Does the slower paced extended calculus course provide a more solid foundation to better prepare students to excel in calculus 2 and future calculus-related courses?

To explore these questions, four years of course data have been collected and analyzed. Academic performance in the subsequent course, Calculus 2, was compared for students from the conventional one-semester course and students from the extended, two-semester, Calculus 1 course.

1.1 Math Placement

Students are placed into their first math course at the University based on the results of a Quantitative Reasoning Assessment, which is based on an industry standard mathematics placement exam. All students entering the university must take that test to determine their first math course placement. This test has two parts, a Basic Algebra portion, and a Pre-Calculus portion. Students who perform sufficiently well to place past College Algebra and College Trigonometry, but do not have sufficiently strong pre-calculus skills to place into the conventional Calculus 1 course, are placed into the first semester of the extended Calculus 1 course.

1.2 Extended, Two-Semester Calculus 1 course

The extended, two-semester, Calculus 1 course is designed to help those students who place into Calculus, but have relatively weak or known gaps in their algebra and trigonometry skills. The course covers all of the standard Calculus 1 content; however, at each point where algebraic or trigonometric concepts are used, the course pauses to review the basic algebra and trigonometric concepts. The students then apply these concepts in continuing to solve calculus problems.

2.0 Problem Statement

Several questions were investigated relating to student performance in a future calculus 2 class. For each of the hypotheses associated with the research questions below, p_1 represents the proportion of students with the desired characteristic in the population of students who took the conventional, single-semester, Calculus 1 course and p_2 represents the proportion of students with the desired characteristic in the population of students who took the extended, two-semester, Calculus 1 course.

Research Question 1: Is there a significant difference in the academic performance in Calculus 2 between those students who successfully completed the conventional Calculus 1 course and those students who successfully completed the extended Calculus 1 course?

> H_0: There is no statistically significant difference between the proportion of students from the conventional, single-semester, Calculus 1 course who passed Calculus 2 and the proportion of students from the extended, two-semester, Calculus 1 course who passed Calculus 2 (H_0: $p_1 = p_2$).
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> H_1: The proportion of students from the conventional, single-semester, Calculus 1 course who passed Calculus 2 is significantly different than the proportion of students from the extended, two-semester, Calculus 1 course who passed Calculus 2 (H_1: $p_1 \neq p_2$).

Research Question 2: Does the slower paced extended Calculus 1 course fail to prepare students to face the rigors of a faster paced conventional Calculus 2 course successfully?

> H_0: The proportion of students from the extended Calculus 1 course who passed Calculus 2 is not significantly lower (equal to or higher) than the proportion of students from the conventional Calculus 1 course who passed Calculus 2 (H_1: $p_2 \geq p_1$).
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> H_1: The proportion of students from the extended Calculus 1 course who passed Calculus 2 is significantly lower than the proportion of students from the conventional Calculus 1 course who passed Calculus 2 (H_1: $p_2 < p_1$).

Research Question 3: Does the slower paced 2-semester Calculus 1 course provide a more solid foundation to better prepare students to excel in Calculus 2?

> H_0: The proportion of students from the extended, 2-semester, Calculus 1 course who passed Calculus 2 is not significantly higher than the proportion of students from the conventional single-semester Calculus 1 course who passed Calculus 2 (H_1: $p_2 \leq p_1$).
>
> H_1: The proportion of students from the extended, 2-semester, Calculus 1 course who passed Calculus 2 is significantly higher than the proportion of students from the conventional, single-semester, Calculus 1 course who passed Calculus 2 (H_1: $p_2 > p_1$).

3.0 Methodology

This study compared student performance in Calculus 2 of students coming from two different methods of instructional delivery for Calculus 1, the first course of a standard calculus sequence. One group of students completed Calculus 1 in the conventional single semester, and the other group of students completed Calculus 1 at a slower pace spread over two semesters with just-in-time review of algebra and trigonometry. These groups of students are not equally paired, since their placement in each group is based on their performance on a math placement test which they took upon college entrance. Students with lower placement test scores are placed into the extended Calculus 1 course, while students with higher placement test scores are placed into the conventional Calculus 1 course.

A total of 1321 students were included in the study, with 1100 students taking the conventional, single-semester, version and 221 students taking the extended, two-semester, version of Calculus 1. For the extended Calculus 1 course sequence, the grade from the final attempt of the second half of the sequence was used as the exiting grade for Calculus 1. For the conventional Calculus 1 course, the grade from the final attempt for Calculus 1 was used as the exiting grade for Calculus 1. In each case, the exiting grade for Calculus 1 was compared to the grade of the same student's first attempt in Calculus 2. This method was used to ensure that the comparison was made between final preparation from Calculus 1 and first attempt at Calculus 2.

The students were divided by which Calculus 1 course delivery type they completed; and individual student grades for Calculus 1 were compared with the same student's grades from Calculus 2. Grades of A, B, or C are considered "passing" grades. Grades of D, F, or W are considered to be "failing" grades, since a grade of C or better is required to move to the next math course in sequence. Calculus 2 grades were evaluated of all students who earned A's, B's and C's in Calculus 1. If a student earned an A or B in Calculus 2, that student was considered to be doing well, so those values were added together. Students who earned D's or F's were considered not to pass the course, so those values were added to each other.

4.0 Results and Discussion

The results of the study can be found in Tables 1 and 2. Table 1 presents the Calculus 2 performance data of those students who completed the conventional, single-semester, Calculus 1 course and Table 2 presents the Calculus 2 performance data of those students who completed the extended, two-semester, Calculus 1 course. Statistically significant differences are printed in bold type and denoted with a superscript asterisk (*).

Table 1. Calculus 2 performance of students who took the conventional, single-semester, Calculus 1

Calculus 1 Grade	Calculus 2 Grade	Number of Students	Percent of Students	% A or B	% A, B, or C	% D or F
A (n=297)	A	176	59.3%	86.2%	94.9%	
	B	80	26.9%			
	C	26	8.8%			
	D	6	2.0%			5.1%
	F	9	3.0%			
B (n=391)	A	89	22.8%	58.6%	**82.4%***	
	B	140	35.8%			
	C	93	**23.8%***			
	D	38	9.7%			17.6%
	F	31	7.9%			
C (n=412)	A	34	8.3%	31.7%	68.0%	
	B	97	23.5%			
	C	149	36.2%			
	D	56	13.6%			32.0%
	F	76	18.4%			
PASS		884	80.4%			
FAIL		216	19.6%			
TOTAL		1100				

Table 2. Calculus 2 performance of students who took the extended, two-semester, Calculus 1 course

Calculus 1 Grade	Calculus 2 Grade	Number of Students	Percent of Students	% A or B	% A, B, or C	% D or F
A (n=37)	A	24	64.9%	91.9%	100%	
	B	10	27.0%			
	C	3	8.1%			
	D	0	0.0%			0%
	F	0	0.0%			
B (n=89)	A	20	22.5%	57.3%	**89.9%***	
	B	31	34.8%			
	C	29	**32.6%***			
	D	5	5.6%			10.1%
	F	4	4.5%			
C (n=95)	A	8	8.4%	37.9%	70.5%	
	B	28	29.5%			
	C	31	32.6%			
	D	13	13.7%			29.5%
	F	15	15.8%			
PASS	A, B, or C	184	83.3%			
FAIL	D or F	37	16.7%			
TOTAL		221				

Comparing the Calculus 2 performance of students who earned A's in Calculus 1 shows that those students who earned an A in the extended Calculus 1 course outperformed their conventional course counterparts in Calculus 2. The data show that 86.2% of the students who earned an A in the conventional Calculus 1 course earned an A or B in Calculus 2, while nearly 92% of the students who earned an A in the extended Calculus 1 course earned an A or B in Calculus 2. Of the students who earned A's in Calculus 1, 5% of the students in the conventional course did not pass Calculus 2 while all of the extended course students passed Calculus 2. While the data seems clear, none of these differences were statistically significant at the 0.05 level.

Of the students who earned B's in the conventional Calculus 1 course, 58.6% earned A's or B's in Calculus 2, while 57.3% of the students who earned B's in the extended course earned A's or B's in Calculus 2. This case was the only instance in which the conventional course students appeared to outperform the extended course students. The difference, however, was not statistically significant.

The pass /fail rates of the students who earned B's in Calculus 1, however, was a dramatically different situation. The proportion of B-level students from the extended Calculus 1 course who passed Calculus 2 was significantly higher than the proportion of B-level students from the conventional Calculus 1 course who passed Calculus 2. In fact, 89.9% of the B-level students from the extended Calculus 1 course passed Calculus 2, while only 82.4% of their B-level conventional Calculus 1 cohorts passed Calculus 2. Similarly, the B-level conventional course students failed Calculus 2 at a rate of 17.6%, while the B-level extended course students had a much lower Calculus 2 failure rate of 10.1%. The extended course B-level students failed Calculus 2 at a statistically significant *lower* rate ($p = 0.0437$) than the conventional course B-level students and the B-level extended course students passed Calculus 2 at a statistically significant *higher* rate ($p = 0.0437$) than the B-level conventional course students.

One specific grade combination was significant. Nearly one-third (32.6%) of the B-level extended course students earned a C in Calculus 2, while only 23.8% of the B-level students from the conventional Calculus 1 course earned a C in Calculus 2. This result ($p = 0.0426$) was statistically significant at the 0.05 level of significance. Since there were no significant differences in the proportions of A's and B's in Calculus 2 based on previous course taken, it seems that the significant difference in the overall passing rate can be attributed to the difference in those Calculus 1 B-level students who earned a C in Calculus 2.

A similar comparison can be made for students who earned C's in their respective Calculus 1 classes. Nearly thirty-two percent (31.7%) of those who earned C's in the conventional course went on to earn A's or B's in Calculus 2. The extended course students who earned C's went on to earn A's or B's at a not significantly higher rate of 38%. The failure rate of the extended course students was also better, with 29.5% failing Calculus 2 compared with 32% of the students who took the conventional course. None of these results, however, were statistically significant at the 0.05 level.

4.1 Analysis of Research Questions

The data collected and presented in Tables 1 and 2 was used to determine answers to the research questions.

Research Question 1: Is there a significant difference in the academic performance in Calculus 2 between those students who successfully completed the conventional, single-semester, Calculus 1 course and those students who successfully completed the extended, two-semester, Calculus 1 course?

The data indicate that, at the 0.05 level of significance, there is no significant difference between the proportion of students from the conventional Calculus 1 course who passed Calculus 2 (0.804 or 80.4%) and the proportion of students from the extended Calculus 1 course who passed Calculus 2 (0.833 or 83.3%). There is no significant difference ($p = 0.6733$) in the proportion of students from either group who pass Calculus 2.

Research Question 2: Does the slower paced extended Calculus 1 course fail to prepare students to face the rigors of a faster paced Calculus 2 course successfully?

The data indicate that, at the 0.05 level of significance, the proportion of students from the extended Calculus 1 course who passed Calculus 2 (0.833 or 83.3%) is not significantly lower than the proportion of students from the conventional Calculus 1 course who passed Calculus 2 (0.804 or 80.4%). This result ($p = 0.3366$) indicates that the slower paced extended Calculus 1 course does not fail to prepare students to face the rigors of the faster-paced Calculus 2 course successfully, as some faculty feared.

Research Question 3: Does the slower paced 2-semester Calculus 1 course provide a more solid foundation to better prepare students to excel in Calculus 2?

The data indicate that, at the 0.05 level of significance, the proportion of students from the extended Calculus 1 course who passed Calculus 2 (0.833 or 83.3%) is not significantly higher than the proportion of students from the conventional Calculus 1 course who passed Calculus 2 (0.804 or 80.4%). While the raw data seems to indicate that students from the extended Calculus 1 course pass Calculus 2 at a higher rate, the difference is not statistically significant ($p = 0.3366$). It appears that the slower paced extended Calculus 1 course, overall, does not better prepare students to excel in Calculus 2.

4.2 Discussion

The finding of no significant difference in overall student performance of the two groups in Calculus 2 indicates that students from either group were equally likely to pass Calculus 2. This result provides evidence to support the claim that students from both groups were relatively equally prepared for Calculus 2. The significance of this finding lies in the fact that both groups were not equally prepared to enter Calculus 1. Those students who were placed into the extended Calculus 1 course were initially deemed to be not prepared to enter calculus directly. There were gaps in their understanding of pre-calculus concepts. The data indicate that the mathematically weaker students who completed the extended Calculus 1 course were able to establish a solid foundation in Calculus 1 and perform at least equally to their counterparts who entered the university more mathematically prepared.

5.0 Conclusions and Recommendations

The data support the conclusion that there is no difference in the proportion of students who pass Calculus 2 based on the type of Calculus 1 course they first completed. Students from both backgrounds appear to pass Calculus 2 in similar proportions. The concern that those students who learn Calculus 1 in the slower model will not be able to pass the faster pace math courses is unfounded. In fact, it appears that students taking the extended Calculus 1 course have a slight advantage in passing Calculus 2 and do so at a higher, but not statistically significantly higher, rate. The two statistically significant results lie in comparing the Calculus 2 pass proportions between students who earned B's in Calculus 1. Approximately 90% of students who earned a B in the extended Calculus 1 course pass Calculus 2 with a C or better, while only 82.4% of the students who earned a B in the conventional single-semester Calculus 1 course earned a C or better in Calculus 2.

The two-semester extended Calculus 1 course seems to be a good way to allow weaker math students an opportunity to "catch up" or improve their math skills and to fill the gaps in their mathematical background before moving to the next course. While it seems to be an effective option for helping students be successful throughout their calculus sequence, the issue of time to graduation is raised. If students take two semesters to complete Calculus 1, is course flow interrupted and is graduation time extended? Can students use summer school to either start the first 2-term course early or catch up with their peers by taking Calculus 2 during the summer? At the authors' institution, students may "catch up" by taking courses during the summer and there are paths to completion within 4 years for most engineering majors. The difficulty comes during the first two years when many courses have a calculus, physics, or statics pre-requisite. At the authors' institution, the pre-requisite for statics is a C or better in Physics I and the pre-requisite for Physics I is a C or better in Calculus 1. Hence, calculus, physics, and statics are on the critical path for graduation in most of the engineering majors. Those engineering majors that do not depend on statics often have other courses which have a physics or calculus pre-requisite. Students in the extended Calculus 1 course must be advised carefully. They often take more general education courses than other engineering students within the first year (since they lack the pre-requisites for physics and the second semester freshman engineering course) and those courses offset general education requirements in later years, thereby creating space in later years to catch up on major-related courses.

Possible questions to investigate in a later study include: Are students who start in the extended Calculus 1 course retained at a higher rate than their counterparts who start in the conventional Calculus 1 course? How do their graduation rates compare? Does the amount of time to graduate differ between the two groups? These concerns center on issues of course sequence and "flow" to graduation. The additional semester of math required in the freshman year may impact the course selections available to the student in the second and third semesters due to pre- and co-requisite requirements, which may affect overall success and retention.

Based on the results of this study, the extended, two-semester, Calculus 1 course appears to be an effective way to facilitate some students' successful path through the Calculus sequence and ultimately through an engineering major to graduation.

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