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Interactive Tutorial System for Linear Circuit Analysis: Impact on Learning and Novel Tutorials

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Abstract

The expansion and evaluation of a step-based tutoring system for linear circuit analysis is described. This system creates its own circuit problems (with variable topologies and element values) for students to solve, along with fully worked example solutions. New tutorial modules have been added, including waveform sketching and Laplace transforms (for a total of 17 released modules). The former involves the sketching of waveforms as a function of time in an interactive web-based interface, where students are required to find the voltage across an inductor when given its current (by differentiating), and other similar problems. The latter involves computation of direct and inverse Laplace transforms from randomly generated functions of various types, where students use an interactive template-based interface to enter their equations for checking. Other important capabilities have been added as well, such as voltage and current division equations, generation of circuit solutions using superposition, and generation of transient circuit problems involving switches. The waveform sketcher is further being adapted to permit sketching of Bode plots from system response functions, and vice versa. A large in-class evaluation was carried out in Fall 2015 with ~70 students to compare the Circuit Tutor system to a widely used commercial publisher-based system. Students were randomly assigned to either use one system for node analysis homework, and the other system for mesh analysis homework, or vice versa. An in-class quiz then compared student performance. Students were also surveyed on their preferences. A large, statistically significant [$t(64) = 3.09$, $p < 0.05$] advantage was found for Circuit Tutor on node analysis of 0.72 standard deviations (average score of 72% for students who used Circuit Tutor, compared to 49% for those who used the publisher system). For mesh analysis, the Circuit Tutor average was 71% vs. 65% for the publisher system, but the difference was not statistically significant [$t(64) = 0.88$, $p = 0.38$]. The larger advantage of this system for node analysis may be due to the fundamentally easier nature of mesh analysis. In the survey, 86% preferred Circuit Tutor and 9% preferred the publisher system, and 94% felt that Circuit Tutor more effectively taught them the topic for which they used it, and 3% felt that the publisher system was more effective. The Circuit Tutor system has now been used by over 2300 students in 54 class sections at eight different colleges and universities, with generally very favorable ratings.

1. Introduction

Linear circuit analysis is a foundational topic for electrical engineering students and frequently comprises the exposure to electrical topics for non-electrical engineers. Optimizing student success in this course is therefore of critical importance. The development of a computer-based tutoring system based on the idea of *step-based tutoring* has therefore been undertaken, where each individual step in a student's work on a problem is accepted and evaluated for correctness before they proceed to the next step of the solution. Such a system requires the creation of special interfaces where students can easily re-draw circuit diagrams as required, enter algebraic equations of various types using pre-defined templates for the various required terms to scaffold their work, enter simplified and matrix equations, interactively sketch waveforms as a function of time and Bode plots as a function of frequency, and enter numerical and other short answers.

Such systems can be more effective and reduce student frustration in comparison to the *answer-based tutors* most often implemented in the past, where only a final numerical answer is checked after the student has already invested considerable time and effort along lines that may not even be correct to pursue. A detailed meta-analysis has shown that the step-based approach typically yields knowledge gains (Cohen *d*-values) of around 0.76 σ (where σ denotes the standard deviation), comparable to those resulting from expert human tutors (0.79 σ) and superior to those of answer based systems (0.31 σ).[1] In a previous, laboratory-based evaluation, this superiority was confirmed with a significant effect size of 1.21 σ and strongly positive impacts on student motivation (0.91 σ) as well.[2,3]

This system further emphasizes the use of worked examples that are exactly isomorphic to the types of problems students are required to solve in the exercises. This approach is supported by the well-known pedagogical importance of learning from examples in the early stages of learning a new cognitive skill.[4-8] Studying worked examples before (and intermingled with) working problems reduces the cognitive load on students and can lead to more effective construction and storage of schemas for working such problems in long-term memory, a key aspect of developing expertise.[6,7] Special strategies (such as color-coding of nodes and equations) are used to minimize the extraneous cognitive load during learning, which can avoid overloading the very limited capacity of working memory.[7] The ability to generate an unlimited number of structurally different problems automatically (unlike most prior tutoring programs in this area[9-14]) allows students to undertake extensive practice, which is necessary to develop the “production rules” necessary to solve problems efficiently.[8]

In the following, recently added modules and capabilities in the program are discussed, which extend the previous work on this system.[2,3,15-18] Recent experiments to evaluate the effect of this system on student learning using randomized, controlled trials and to measure student satisfaction with the tool are then described.

2. Overview of Random Problem Generation

As described previously, this system creates its own circuit problems from scratch using a sophisticated three-stage algorithm designed to produce problems that are very similar to the problems typically found in linear circuit analysis textbooks.[2,3,15-18] These problems are presented as diagrams laid out on a square grid (for convenience in avoiding graphical interferences), which can however represent any planar circuit of reasonable size (on a grid of up to 5 × 5 squares). The usable grid size and numbers of nodes, meshes, and each type of circuit element (independent voltage & current sources, dependent sources of both types, resistors, capacitors, inductors, and normally-closed and normally-open single-pole switches) can all be specified (consistent with the relationships that must hold among these quantities). The number of branch voltages, branch currents, branch powers (absorbed or supplied), and non-branch voltages (that do not appear across a single circuit element) that the student is to find can also be specified (down to the type of element on which they should be randomly placed). Special options are available to request a specific number of “floating supernodes” (those not containing the reference node) or to have or lack an internal current source (which would necessitate use of supermeshes in solving via mesh analysis). The number of voltage sources in series and current sources in parallel can be limited to any desired values, and there is an option to allow or prohibit passive elements of the same type in series or parallel. The occurrence of “redundant” sources and passive elements (such as those in series with a current source, or in parallel with a voltage source) can be allowed or prevented. Circuits that require wye or delta transformations to be fully simplified can be rejected. The program ensures that all generated circuits are fully solvable by both node and mesh analysis (both before and after action of any switches that are included). With the above capabilities, circuits varying from extremely simple to very complex can be generated, so that any desired level of difficulty can be achieved.

An example of a randomly generated problem is shown in Fig. 1. (Note: Due to their size, all figure in this paper are shown in an Appendix.) This circuit contains a switch (to create a transient problem) and has been automatically solved at $t = \infty$ by the computer, where it replaced the capacitor by its steady-state equivalent of an open circuit. An expansion of this capability will be used to generate full transient problems in the near future.

3. New Problem Types and Solution Methods

Previously this system solved both DC and steady-state AC (phasor) circuit problems via node or mesh analysis, including features such as supernodes, supermeshes, and dependent sources. All steps are shown in detail in the solution process. This approach is quite different from numerical circuit solvers such as PSPICE, which use a method called modified nodal analysis that is not typically taught to beginning students, and which show only final computed answers (not the steps involved in the algebraic solution). The system has now been extended to use other important methods, such as single node-pair and single loop (voltage division and current division) analysis and superposition. All such capabilities are available for both DC and AC circuits, as complex numbers are used internally to represent all voltage sources, current sources, and impedances (even if DC, in which case the imaginary parts are zero). An example of a solution using these methods is shown in Fig. 2 (see Appendix). The solution involves the successive inactivation of each source, followed by a solution using voltage division or Ohm's law as appropriate. A similar approach is used for current division (where the current division is explained using both conductance and resistance-based approaches, or admittance and impedance-based approaches in AC cases). In exercises, students will first edit the circuits appropriately using a graphic circuit editor[16] to remove one source or the other, then enter their proposed solutions using appropriate equation templates that they fill in.

Recently added modules cover the mathematics of direct and inverse Laplace transforms. These modules are purely web-based for easy accessibility. The functions to be transformed or inverse transformed are chosen randomly, with a random number of poles and zeroes and both underdamped and overdamped cases for the inverse transforms. The mathematics are explained in a high level of detail in the examples, as illustrated in Fig. 3 (see Appendix). Students enter solutions for exercises using a template-based system where they choose the appropriate types of terms and then fill in the required values. Multiple steps such as the partial fraction expansions are checked in keeping with the step-based approach.

Other recently added modules include exercises where students must sketch one electrical quantity as a function of time, given a graph of a different quantity. In the Basic Electrical Quantities tutorial, these quantities include charge, current, voltage, power, and energy. The waveforms can be piecewise constant, ramps, parabolas, sinusoids, or exponentials. Details of the waveforms, such as the number and type of piecewise segments that are present, are varied randomly to assure a good variety of problems, as well as the magnitudes of the quantities (exercising skills with metric prefixes and unit conversions). Some of the problems involve differentiating or integrating one quantity to find another. In the similar Capacitors and Inductors tutorial, the quantities include charge, magnetic flux, current, and voltage of capacitors or inductors (problems involving energy and power will also be added). An interactive interface is used for the sketching process, and students are given immediate feedback about the correctness of their sketches. Detailed example solutions are provided for problems that are isomorphic to the exercises.

Another module under development explains the drawing of Bode plots for given transfer functions, which are as usual randomly generated. (This module will be extended further to cover extraction of a transfer function from a Bode plot.) A portion of an example solution of this type is shown in Fig. 4 (see Appendix). Again, an attempt is made to explain every aspect of the required mathematics, including details of how to take the magnitude of a complex-valued function with multiple terms.

4. Evaluation of Learning Effectiveness

Both laboratory-based and classroom-based evaluations documenting the high effectiveness of this system in promoting student learning were previously reported, using randomized, controlled experiments.[2,3,15] An additional such experiment was performed in Fall 2015 in one section of EEE 202 (Circuits I) at ASU with about 70 students. The goal was to compare the learning effectiveness of this system in comparison with a widely-used publisher-based commercial homework system (called "System X" in the following). The commercial system mainly uses algorithmic versions of problems in the textbook (where a few element values are randomly changed, but not the circuit layout), and provides answer-based tutoring where the student is usually just told whether or not their final (usually numerical) answer is correct. In a few cases, intermediate results are requested. Students were randomly assigned to either use Circuit Tutor or System X for DC node analysis, and the opposite system for DC mesh analysis (Group A), or vice versa (Group B). The Circuit Tutor exercises included both writing the relevant types of equations, and a separate module where they had to be fully solved. Circuit Tutor provided corresponding examples, though students were not required to view them. The number and types of problems assigned in System X were designed to be very similar to the number and type of problems students had to work in Circuit Tutor, though in the latter case, they may need to work more if they choose to give up on a problem or make too many errors. System X was configured to allow four attempts to get the correct answers, after which the solution was shown. The homework in the two systems was assigned following the normal lecture coverage of that material and assigned readings on it from the textbook.

Immediately after the due date for the assignment, students were given a post-test in the form of a paper quiz on both topics, where they had to write and in some cases solve the relevant equations. Grading was done "blindly" without knowledge of the group assignments. No pre-test was used on this topic, but random assignment makes it quite unlikely that the two groups were significantly different in their prior knowledge. Their scores on a conceptual pre-test on electrical fundamentals (the DIRECT 1.0 concept inventory[19]) were nearly the same (53.2% for Group A and 54.8% for Group B), as were their overall course scores (which included many other topics), 68.5% vs. 72.0%, respectively. To gauge student opinion on the relative merits of the two systems, all students were requested to complete an anonymous survey through Blackboard (all of the students had used both systems for one half of their assignment).

Student performance was compared using several metrics, including their score on the homework assignment itself, the post-test scores of both groups, and the survey results. The quantitative results are summarized in Table I. Circuit Tutor showed a statistically significant advantage for the post-test scores on node analysis [$t(64) = 3.09$, $p < 0.05$] with an effect size (Cohen *d*-value) of 0.72σ. For mesh analysis, the difference was not statistically significant [$t(64) = 0.88$, $p = 0.38$], which may reflect the fundamentally easier nature of that topic (both groups had relatively high averages). The survey results showed a very strong preference for Circuit Tutor and a strong belief that it taught them more effectively than System X. A typical student comment was "I liked Circuit Tutor more because I could do a ton of problems. I liked that even if I couldn't figure it out, I could 'give up'; and it would thoroughly explain how to do everything so I could understand what I did wrong and then do a new problem and try to get that right. I seem to retain more of the content when I am doing this one. I have trouble with [System X] because, if I have trouble with a problem, the hints do not explain what I am doing wrong. It's really frustrating because I could be 2 or 3 wrong attempts in and I do not know what I'm doing wrong." These survey results strongly confirm the observation in a similar experiment in Fall 2014 (which however lacked the post-test) that students assigned to use System X would often voluntarily use Circuit Tutor on the same topic (for no additional credit), but students assigned to Circuit Tutor *never* voluntarily used System X on the same topic (though it was made available and students were informed of the option).[17]

5. Student Usage and Evaluations

To date, this system has been used by about 2300 students in 54 class sections at eight different colleges and universities, including an elite private university, medium and large public state universities, a primarily bachelor-level private university, a small private college, an historically black urban university, and several community colleges. The goal is to establish the utility of the system in a wide variety of settings with different types of student populations.

To evaluate the overall opinions of students about the system, a one-question survey is conducted at the conclusion of each tutorial, in which students are asked to state whether the tutorial was "very useful," "somewhat useful," "not very useful," or "a waste of time" for learning the material. For the most recent year Spring 2015-Fall 2015, the responses in each category (combining all tutorials) were 65%, 26%, 4%, and 5%, respectively, for a total of 91% favorable (very or somewhat useful). The ratings have decreased a little as many more tutorials were added, but have generally remained consistent, showing an overall high level of satisfaction.

Table I. Results of Classroom-Based Experiment in Fall 2015.

System used	Circuit Tutor	System X
Mean post-test score–node analysis (std. dev.)	72%(24%)	49%(33%)
Mean post-test score–mesh analysis (std. dev.)	71%(25%)	65%(31%)
Mean node/mesh HW score	79%	69%
Preferred system in question	86%	9%*
Felt system in question taught more effectively	94%	3%*

*Remaining students rated both systems equal

A more detailed anonymous survey is conducted at the end

Table II. Percentage of Favorable Scores on End-of-Semester Survey by Institution

Institution	Useful & well designed	Appropriate. difficulty & coverage	Prefer to conventional homework	Overall	*N*
Arizona State Univ.	80.0%	78.6%	78.5%	79.0%	841
Morgan State Univ.	78.1%	70.1%	79.5%	75.9%	56
Univ. of the Pacific	69.4%	71.1%	53.3%	64.6%	45
Messiah College	87.1%	91.4%	82.8%	87.1%	58
Univ. of Notre Dame	91.8%	91.9%	87.9%	90.5%	31
All (combined)	80.1%	78.7%	77.8%	78.9%	1031

of each semester, in which users are asked a total of 12 questions on a 4-point Likert scale to determine 1) if the tutorials are useful and well designed; 2) if the difficulty and coverage are appropriate, and 3) if students prefer the tutorials over conventional homework. The percentages of favorable responses in each category of question are shown in Table II for each institution whose students took the survey, and in total. The results show a high level of satisfaction in general, around 80%. In particular, good results are obtained at a wide variety of different types of institutions, suggesting that the software should be broadly useful in many different settings. The results were a little lower at the University of the Pacific, which may be because the tutorials were used there in combination with several other types of homework on the same topics and may therefore have been viewed as somewhat redundant. In most cases they have been used as a replacement for some of the conventional homework (for the topics currently covered by the system). The students at Morgan State University tended to feel more than others that the tutorials were somewhat too long and difficult.

6. Conclusions

A step-based tutorial system to teach linear circuit analysis has been expanded and evaluated, adding new capabilities such as switches, voltage and current division analysis and superposition, generation of Bode plots, and new tutorials involving waveform sketching and Laplace transforms. A key unique feature of the system is its ability to generate an unlimited supply of both problems and worked examples that differ in both circuit topology and element values. A new randomized, controlled classroom-based experiment yielded evidence for substantial superiority of this system in learning effectiveness over a mature, commercial publisher-based system for node analysis and comparable results for mesh analysis. Students strongly preferred the new system and felt that it helped them learn better. Overall student satisfaction has remained high as the scope of the system has been expanded to cover the new topics. Future plans include additional expansion to cover all of the topics in a traditional two-semester circuits class as well as additional rigorous evaluations to further measure its effectiveness.

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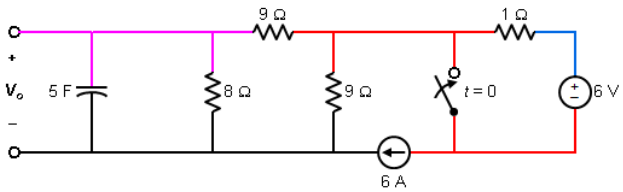
Appendix (Figures)



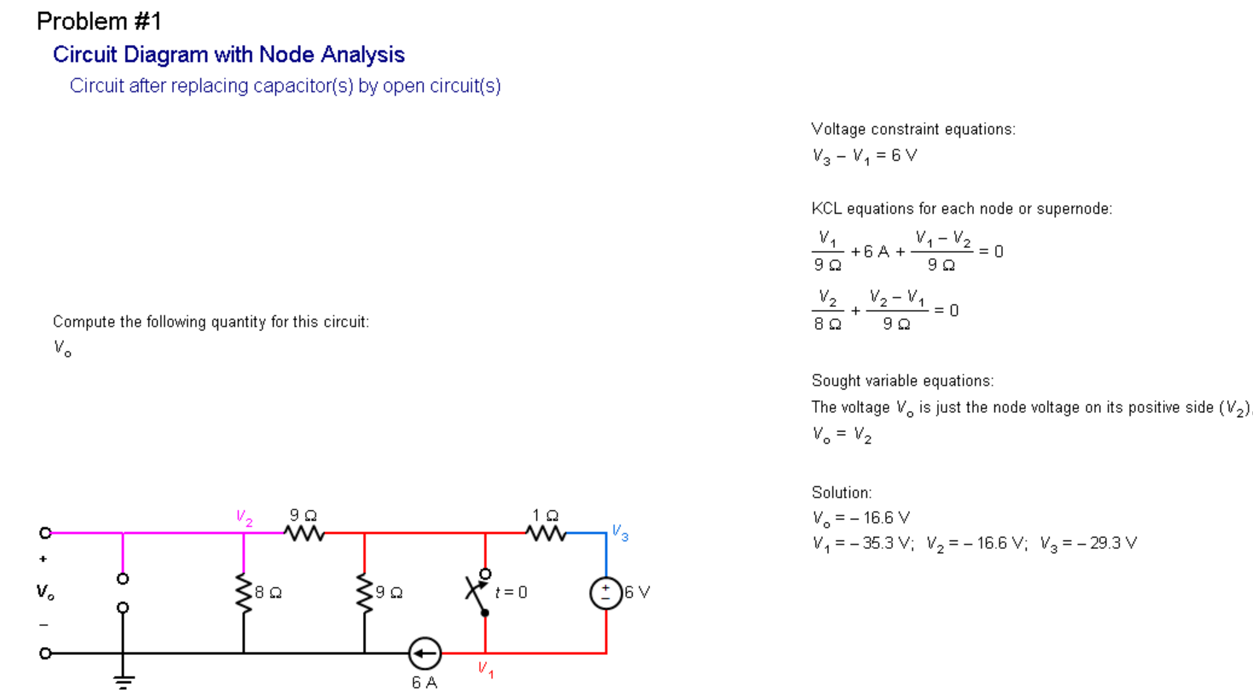


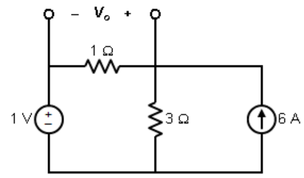
Fig. 1. Screen shot of a randomly generated transient circuit containing a switch that closes at $t = 0$ (top portion). The circuit is solved in the bottom portion at $t = \infty$, automatically treating the capacitor as an open circuit and assuming the switch is closed.

Original Circuit with All Sources Active

Circuit Diagram

Compute the following quantity for this circuit:

V_o

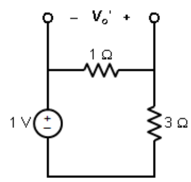


Circuit with Only the 1 V Independent Source Active

Circuit Diagram

Compute the following quantity for this circuit:

V_o'



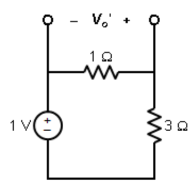
Circuit with Only the 1 V Independent Source Active

Circuit Diagram with Single Loop Analysis

(Original circuit, no series–parallel simplifications possible while preserving sought variables)

Compute the following quantity for this circuit:

V_o'



Sought variable equations:

In a series circuit, the voltage provided by the source is divided among the resistances.

The voltage across each resistance is the source voltage multiplied by the ratio of the resistance in question to the sum of all the resistances in series.

Because the polarities of the voltage source and the sought voltage have opposite senses (the positive side of the source is connected to the negative side of the sought voltage), the source voltage appears with a negative sign.

$$V_o' = -1\text{ V}\,\frac{1\,\Omega}{1\,\Omega + 3\,\Omega} = -0.25\text{ V}$$

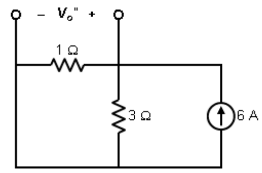
Fig. 2. Screen shot of an example solution of a randomly generated circuit using superposition and single loop/single node-pair analysis (continued on next page.)

Circuit with Only the 6 A Independent Source Active

Circuit Diagram

Compute the following quantity for this circuit:

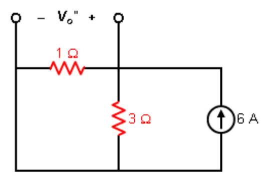
V_o''



Circuit with Only the 6 A Independent Source Active

Series–parallel pre–simplification

Circuit prior to simplification; first set to be combined is highlighted in red.



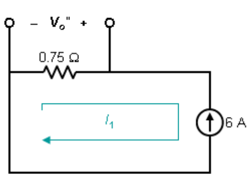
Circuit with Only the 6 A Independent Source Active

Circuit Diagram with Single Loop Analysis

Circuit after combining the 3 Ω and 1 Ω resistors in parallel (all series–parallel simplifications that preserve the sought quantities now completed)

Compute the following quantity for this circuit:

V_o''



Sought variable equations:

We define the current I_1 to be that flowing in a clockwise direction, as shown on the diagram. Note that I_1 can be positive or negative in general. In this case, the value of I_1 is fixed by the value of the current source. Given that the current source arrow points in the opposite direction to that of I_1, we have

$I_1 = -6$ A

Using the passive sign convention, the voltage V_o'' across the 0.75 Ω resistance is just that resistance multiplied by the net current entering its positive side. This current is just the negative of current I_1, because I_1 enters the negative side of voltage V_o''.

$V_o'' = -I_1(0.75\ \Omega)$

Solution:

$V_o'' = 4.5$ V

$I_1 = -6$ A

Original Circuit with All Sources Active (Combining Results from Superposition)

Circuit Diagram

Compute the following quantity for this circuit:

V_o

$V_o = V_o' + V_o'' = -0.250\ \text{V} + 4.50\ \text{V} = 4.25\ \text{V}$

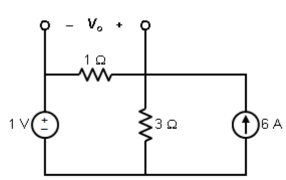


Fig. 2 (continued). Screen shot of the solution of a randomly generated circuit using superposition and single loop/single node-pair analysis.

The goal is to find the inverse Laplace transform of the function,

$$\mathbf{F(s)} = \frac{5(\mathbf{s}+40)}{(\mathbf{s}+10)(\mathbf{s}^2+10\mathbf{s}+29)}.$$

The first step is to factor the quadratic terms in $\mathbf{F(s)}$. For the general quadratic equation $a\mathbf{s}^2 + b\mathbf{s} + c = 0$, the roots are given by:

$$\mathbf{s}_{1,2} = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}},$$

which implies

$$(\mathbf{s}-\mathbf{s}_1)(\mathbf{s}-\mathbf{s}_2) = 0.$$

For $\mathbf{s}^2 + 10\mathbf{s} + 29 = 0$, we have

$$\begin{aligned}\mathbf{s}_{1,2} &= \frac{-(10)}{2} \pm \sqrt{\left(\frac{10}{2}\right)^2 - 29} \\ &= -5 \pm j2.\end{aligned}$$

So,

$$\mathbf{s}^2 + 10\mathbf{s} + 29 = (\mathbf{s}+5-j2)(\mathbf{s}+5+j2).$$

Therefore, $\mathbf{F(s)}$ can be written in factored form as

$$\mathbf{F(s)} = \frac{5(\mathbf{s}+40)}{(\mathbf{s}+10)(\mathbf{s}+5-j2)(\mathbf{s}+5+j2)}.$$

The inverse Laplace transform is obtained by expressing $\mathbf{F(s)}$ in a partial fraction expansion:

$$\mathbf{F(s)} = \frac{k_0}{\mathbf{s}+p_0} + \frac{\mathbf{k}_1}{\mathbf{s}+\mathbf{p}_1} + \frac{\mathbf{k}_1^*}{\mathbf{s}+\mathbf{p}_1^*},$$

where $p_0 = 10$, $\mathbf{p}_1 = 5 - j2$, and $\mathbf{p}_1^* = 5 + j2$.
Note: * denotes the complex conjugate of a complex number and $-p_0, -p_1$, etc. are the poles of $\mathbf{F(s)}$.

The values of $\mathbf{k}_i$'s are obtained as follows:

$$\mathbf{k}_i = (\mathbf{s}+p_i)\mathbf{F(s)}\Big|_{\mathbf{s}=-p_i}.$$

Therefore,

$$\begin{aligned}k_0 &= \cancel{(\mathbf{s}+10)}\,\frac{5(\mathbf{s}+40)}{\cancel{(\mathbf{s}+10)}\,(\mathbf{s}+5-j2)(\mathbf{s}+5+j2)}\Bigg|_{\mathbf{s}=-10} \\ &= \frac{5(-10+40)}{(-10+5-j2)(-10+5+j2)} \\ &= \frac{5(30)}{(-5-j2)(-5+j2)} \\ &= 5.1723\end{aligned}$$

Fig. 3. Screen shot of a portion of the computer-generated example solution of an inverse Laplace transform solution of a randomly generated function. The remainder is not shown for brevity but is similarly detailed.

Problem

Draw a Bode magnitude plot for the transfer function

$$\mathbf{H}(j\omega) = \frac{4000\,(j\omega + 2)}{j\omega\left[(j\omega)^2 + 8j\omega + 400\right]}.$$

Solution:

***Step 1*: Rewrite the transfer function in standard form.**
Note: In standard form, the constant of each term is unity.

$$\mathbf{H}(j\omega) = \frac{4000\,(j\omega + 2)}{j\omega\left[(j\omega)^2 + 8j\omega + 400\right]}$$

$$= \frac{4000 \cdot 2 \cdot \left(\frac{j\omega}{2} + 1\right)}{400 \cdot j\omega\left[\frac{(j\omega)^2}{400} + \frac{8}{400}j\omega + 1\right]}$$

$$= \frac{20\left(1 + \frac{j\omega}{2}\right)}{j\omega\left[1 + 0.02j\omega + \left(\frac{j\omega}{20}\right)^2\right]}. \qquad (1)$$

***Step 2*: Identify the terms making up the transfer function.**
This transfer function has the following terms:

- A constant coefficient with an value of 20 = 20 log(20) dB = 26 dB
- A pole at the origin
- A simple zero at a break frequency of ω = 2 rad/s
- A pair of complex conjugate poles at a break frequency of ω = 20 rad/s

Hint: Poles are terms in the denominator involving $j\omega$, and zeros are terms in the numerator involving $j\omega$.

***Step 3*: Find the complex magnitude of the transfer function.** Rules for Making Bode Plots

Details of Finding |H($j\omega$)|

The magnitude of a complex number $\mathbf{z} = x + jy$ is $|\mathbf{z}| = \sqrt{x^2 + y^2}$ (the distance of the point z in the complex plane from the origin). Thus, $|\mathbf{z}|^2 = x^2 + y^2$. As x and y are real numbers, the definition implies that a complex magnitude can NEVER be negative! Further, $|\mathbf{z}_1/\mathbf{z}_2| = |\mathbf{z}_1|/|\mathbf{z}_2|$ and $|\mathbf{z}_1\mathbf{z}_2| = |\mathbf{z}_1||\mathbf{z}_2|$, which are easily seen from the procedures for division and multiplication of complex numbers in polar form. Therefore, finding the magnitude of $\mathbf{H}(j\omega)$ requires that we first compute the magnitude of each term and then multiply or divide the magnitudes of those terms as appropriate. For each of the following terms we first evaluate $j^2 = -1$. Then we identify the sum of all numbers not multiplying j as the real part, and the sum of all terms that multiplying j as the imaginary part, by definition.

Term	Real Part	Imaginary Part	Magnitude
$j\omega$	0	ω	ω
$1 + \frac{j\omega}{2}$	1	$\frac{\omega}{2}$	$\sqrt{1 + \left(\frac{\omega}{2}\right)^2}$
$1 + 0.02j\omega + \frac{(j\omega)^2}{400}$	$1 - \frac{\omega^2}{400}$	0.02ω	$\sqrt{\left(1 - \frac{\omega^2}{400}\right)^2 + (0.02\omega)^2}$

Fig. 4. Screen shot of a portion of the computer-generated example solution of a randomly generated Bode plot problem (continued on next page).

The complex magnitude of the transfer function is

$$|\mathbf{H}(j\omega)| = \frac{20\sqrt{1+\left(\frac{\omega}{2}\right)^2}}{\omega\sqrt{\left(1-\frac{\omega^2}{400}\right)^2+(0.02\omega)^2}}. \quad (2)$$

***Step 4*: Approximate the transfer function between each pair of breakpoints.**

The next step is to approximate the above expression for values of ω between $-\infty$ and the first breakpoint, between the first and the second breakpoint, and so forth, and from the last breakpoint to $+\infty$. In this case, the break frequencies are 2, and 20 rad/s in order of increasing frequency. In each range, we assume in the spirit of an asymptotic approximation (whether or not it is accurate to do so) that ω is much bigger than the breakpoint to its left and much smaller than the breakpoint to its right.

The first range of ω is from $-\infty$ to 2 rad/s. In this range, we assume $\omega \ll 2$ rad/s. Therefore, $\omega / 2 \ll 1$, $\omega / 20 \ll 1$, and

$$|\mathbf{H}(j\omega)| \approx \frac{20\sqrt{1+\cancel{\left(\frac{\omega}{2}\right)^2}}}{\omega\sqrt{\left(1-\cancel{\frac{\omega^2}{400}}\right)^2+\cancel{(0.02\omega)^2}}} = 20\omega^{-1}$$

$$= 20\log(20\omega^{-1})\text{ dB} = 20\log(20)\text{ dB} - 20\log\omega\text{ dB} = 26\text{ dB} - 20\log\omega\text{ dB}.$$

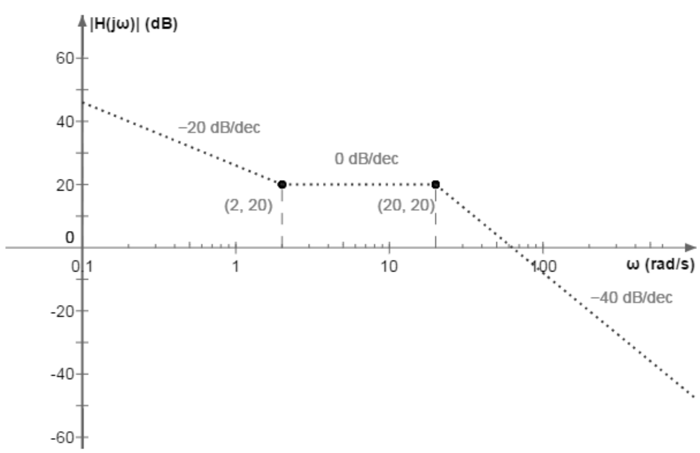
Note: the diagonal lines indicate that a term is being being neglected, not that it is being cancelled.In the above we use the fact that $\log ab = \log a + \log b$. Note that $\log(\omega^{-1}) = -\log\omega$, because $\log x^n = n\log x$, where n is any positive or negative number. We can rewrite this expression in the form $y = mx + b$, where $y = |\mathbf{H}(j\omega)|$, $x = \log\omega$, $m = -20$ dB/dec and $b = 26$ dB, or

$$|\mathbf{H}(j\omega)| = \left(-20\,\frac{\text{dB}}{\text{dec}}\right)\log\omega + 26\text{ dB}.$$

Note that the unit of slope is dB/dec because incrementing $\log\omega$ by one corresponds to incrementing ω by a factor of 10, which can be seen as follows: $\log(10\omega) = \log 10 + \log\omega = 1 + \log\omega$.

(LARGE PORTIONS OMITTED HERE)

Using this table, we can sketch the asymptototic approximation to the Bode magnitude plot from $-\infty$ to $+\infty$ as shown below. Specifically, we plot a point for the magnitude of $|\mathbf{H}(j\omega)|$ at each break frequency and connect those points by straight lines. Then we use the slope in each end region to continue the sketch into those regions.



(ADDITIONAL PORTIONS OMITTED HERE)

Fig. 4 (continued). Screen shot of additional portions of the example solution of a Bode plot problem.