If We Can't Model a Cantilevered Beam, What Can We Model? Helping Students Understand Errors in Vibration Experiments and Analyses

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If we can't model a cantilevered beam, what can we model? Helping students understand errors in vibration experiments and analyses

Students often view both analytical results and experimental results with supreme confidence without critically evaluating the assumptions behind them. In the Mechanical Vibrations course at Rose-Hulman Institute of Technology, lab experiences have been developed to help address this deficiency in students' understanding of models, experiments, and their limitations. In the first lab, students are required to determine the first natural frequency of a cantilevered beam experimentally using several different approaches and then compare their findings to analytical results. The lab has a final project involving an experimental modal test and the creation of a finite element model of a structure of the students' choosing. Students are required to propose explanations for the differences in the results from the test and the finite element model. Assessment results show that students have developed a much more sophisticated understanding of analysis and testing as a result of these experiences, and by the end of the course, they use appropriate technical terminology when discussing the differences between test and analytical results.

Background

According to the National Research Council report *How People Learn: Brain, Mind, Experience, and School* [1], one aspect of effective learning is its durability: does the learning have long-term impact in the ways it influences other kinds of learning or performance? A key conclusion in this report on the concept of durability is that it is essential for a learner to develop a sense of when what has been learned can be used, i.e., the conditions of application. Failure to transfer is often due to learners' lack of this type of conditional knowledge. If the problems examined in a vibrations course are always presented in the context of idealized mass, stiffness, and damping elements—that is, looking nothing like a realistic system—then it will be difficult for students to apply the concepts discussed in the course. Similarly, if finite element modeling is never coupled with an experimental test of the item being modeled, students will potentially lack the appropriate skepticism of finite element analysis and its associated assumptions.

Vibrations courses often do not have a dedicated lab associated with them, and the only exposure students have to vibrations experiments is in more general laboratory courses on engineering measurements [2, 3]. Some schools do have vibrations and structural dynamics courses with integrated lectures and laboratories [4-9], but often the purpose of these labs is to illustrate the main concepts in the lectures with hands-on laboratory exercises or they primarily involve MATLAB/Simulink modeling and simulation. This approach can help motivate the course material but can also afford students the opportunity to see differences between theory and measured experimental results. This is critical in helping students understand that models have assumptions and limitations.

At the University of Toledo, the laboratory experiments for a Mechanics and Vibrations Laboratory have been redesigned to transform the learning process from subject-based learning to problem-solving learning [6]. This course has Mechanical Vibrations as a prerequisite, and therefore the lab does not enhance the vibrations course. One goal for the lab is to provide students with more hands-on experiences and to challenge them by requiring the procedure for each laboratory experiment to be designed and carried out by each group of students. The authors state that due to the number of students and the limited number of lab sessions, it is difficult to provide the students with real hands-on experience with the instrumentation and lab setup as desired.

It is clear that experimental vibration analysis is an important tool for identifying dynamic properties of mechanical structures, and yet this important tool is not a part of many vibrations courses. Experimentation is also critical to helping students understand that models and reality are always different, although some models are better than others. Poincaré [10] stated it this way: "experiment is the sole source of truth." To help bridge this gap between experimentation and theory, and to equip students with practical skills in experimental vibrations, starting in the 2006-2007 academic year, a lab was added to the Mechanical Vibrations course at Rose-Hulman. The original objective of the lab was for the students to design and conduct a modal test of a structure of their choice. Details of this lab experience have been discussed in a previous paper [11]. This lab has evolved over the years to give students the experience not only of performing an experimental modal test but also of performing a finite element analysis of the same structure. At the conclusion of these experiences, students are asked to provide plausible explanations for the differences between the test and the analytical results.

Goals and methods

The goals of this study were to determine if the current laboratory activities

1. improve students' self-identified experience in performing an experimental modal test,
2. improve students' self-identified experience in analytical modal analysis using a commercial finite element analysis package, and
3. improve students' ability to use appropriate technical language when discussing the differences between test and analytical results.

To accomplish these goals, the lab experience starts with a simple model/test correlation of a cantilevered beam and culminates in a weeks-long final model/test project. In the first lab, students working in teams are required to determine the first natural frequency of a cantilevered beam experimentally using several different approaches and compare their findings to analytical results. All test beams have nominally identical dimensions and material properties, but the accelerometers the teams use have slightly different masses. The results from all teams are collected and presented to the class. The class then discusses these results, and students are asked to explain the differences between the teams' results. Cantilevered beam testing has been discussed in other papers (e.g., see [12]), but not with the objectives discussed in this paper. The lab concludes with a multi-week team project involving an experimental modal test and the creation of a finite element model of a structure of their choosing. Students are required to propose explanations for the differences between the experimental and finite element results.

To determine if we achieved our goals, an anonymous survey was distributed at the beginning of the first lab day and at the end of the last lab day. The survey asked students to rate their experience using finite element analysis and their experience with experimental vibrations. Students were also asked to discuss which they would trust more, experimental results or finite element results, and they were asked to provide possible explanations for the differences between the two.

Description of the equipment

Each team was assigned a "vibrations kit," shown in Figure 1, that included an LDS PHOTON II data acquisition unit, an instrumented hammer, and two accelerometers.

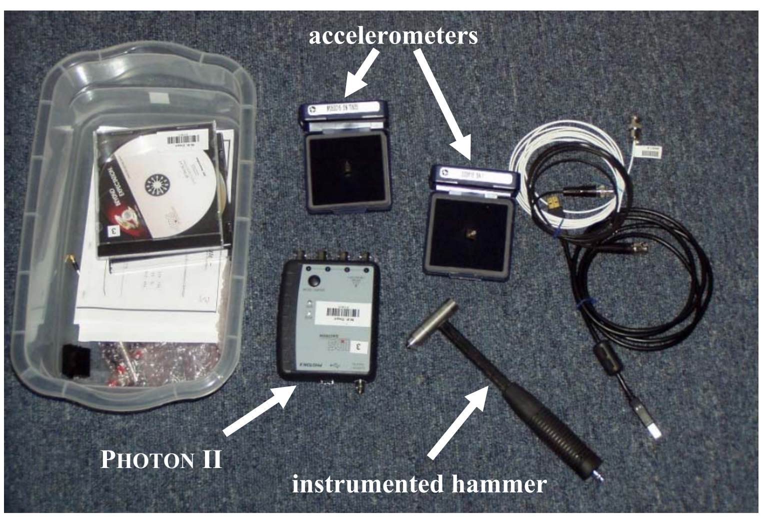


Figure 1. The contents of a vibrations kit used in the lab.

Each team also received an accompanying Lenovo ThinkPad laptop running Windows 7 that their PHOTON II unit connected to via USB, as shown in Figure 2. Frequency response and time history data were collected using RT Pro software loaded on the laptops.

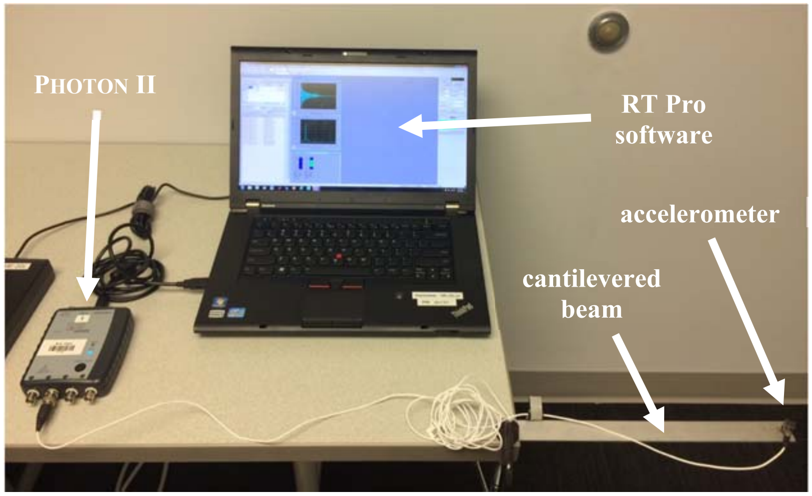
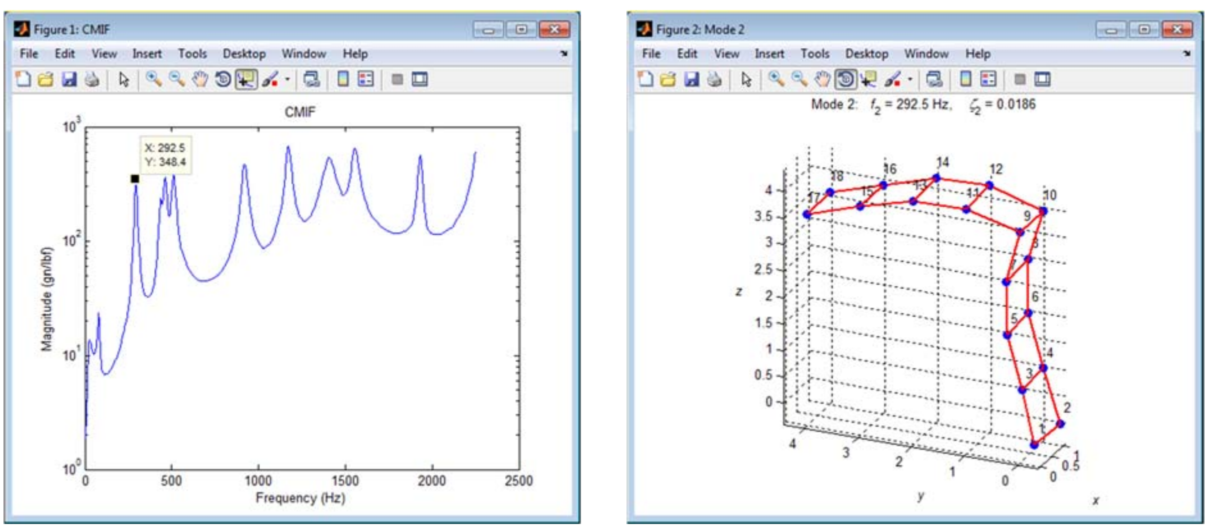


Figure 2. A PHOTON II device interfaced with a laptop running RT Pro to collect data for the cantilevered beam experiment.

Students determined the experimental natural frequencies and mode shapes of their chosen structure for the final project using a MATLAB-based modal analysis program created in-house. The program, named EMAP (*E*asy *M*odal *A*nalysis *P*rogram), was designed to make it easy for students to load their frequency response data and structure geometry, identify natural frequencies by peak picking, and extract the corresponding mode shapes. Figure 3 contains a few screenshots of EMAP in use. Teams created and analyzed their accompanying finite element models in SOLIDWORKS.



(a) (b)

Figure 3. Screenshots illustrating the use of EMAP for modal analysis of an L-bracket. In (a) the natural frequency of the second nontrivial mode is being identified by peak picking. The corresponding mode shape is shown in (b).

Results from the cantilevered beam experiment

Each student team was given a rectangular aluminum beam. They clamped it to a table and attached an accelerometer to its free end, similar to the setup shown in Figure 2. The beam was plucked to measure the acceleration time history of the beam's tip. Students determined the beam's first natural frequency using a lumped mass model, a lumped mass model that included the mass of the accelerometer, a finite element model, and the experimental data by peak picking in the frequency domain and by using the period from the time history. Table 1 shows the results from one section of the course.

Table 1. The natural frequency of a cantilevered beam's first mode determined by nine lab teams using various analytical and experimental approaches.

Team	Analytical natural frequency (Hz)			Exp. natural frequency (Hz)	
	Lumped mass	Lumped mass with accelerometer mass	Finite element (no accelerometer mass)	Time domain	Frequency domain (peak picking)
1	29.03	24.94	28.10	21.34	21.41
2	29.02	24.40	28.10	21.06	20.94
3	29.04	24.42	28.10	22.12	22.19
4	29.03	24.94	28.10	21.20	21.09
5	29.03	24.94	28.10	20.14	20.00
6	29.03	25.21	28.10	21.05	21.09
7	29.03	24.41	28.10	21.45	21.41
8	28.42	23.94	28.10	21.48	21.41
9	29.02	24.43	28.10	21.48	21.09

After all teams had completed their calculations, the results were presented to the class and discussed. The points brought out during the class discussion included the following:

1. *Differences in experimental results* — The results from the time- and frequency-domain methods using the experimental data were very similar, but in general, they were not identical. This was because the peak picking approach depended on the frequency increment in the spectrum magnitude plot. The period from the time history assumes a single-degree-of-freedom system, but other modes are actually present. There is also uncertainty in determining the period. The difference between the results of the various teams was most likely due to the different masses of the accelerometers and the different ways the students clamped the beams.
2. *Differences in analytical approaches* — For the lumped mass approaches, the natural frequency f_n is computed according to

$$f_{\mathrm{n}} = \frac{1}{2\pi}\sqrt{\frac{k_{\mathrm{eq}}}{m_{\mathrm{eq}}}}\,, \tag{1}$$

where the equivalent stiffness

$$k_{\mathrm{eq}} = \frac{3EI}{L^3} \tag{2}$$

depends on the beam's elastic modulus E, area moment of inertia I, and length L. Considering both the beam's mass, m_{beam}, and the mass of the accelerometer, m_{accel}, the equivalent mass in Eq. (1) is

$$m_{\mathrm{eq}} = 0.23\, m_{\mathrm{beam}} + m_{\mathrm{accel}} \tag{3}$$

Neglecting the mass of the attached accelerometer, the differences in the teams' results (in the "Lumped mass" column of Table 1) can be attributed to some teams having actually measured the beam geometry rather than using the nominal values and some teams using slightly different values for the material density and/or elastic modulus. When the accelerometer mass was factored in, the differences in results were due to the different masses of the accelerometers. Every team used the same finite element model because we had not yet discussed how to perform a finite element analysis at this point in the class. The differences between the lumped mass approach and the finite element analysis are due to the finite element model not treating the beam as a single-degree-of-freedom system.

3. *Differences between analytical and experimental results* — The analytical results were approximately 16% higher than the experimental results. Students were required to investigate whether reasonable uncertainty, ±5%, in the dimensions or the material properties could explain this difference; they discovered these uncertainties were not sufficient to explain the difference. Eventually, the class recognized that the analytical methods all assumed a fixed boundary condition, whereas in actuality, the boundary condition was not perfectly fixed—both the clamp and the table had some compliance.

Results from the experimental modal analysis

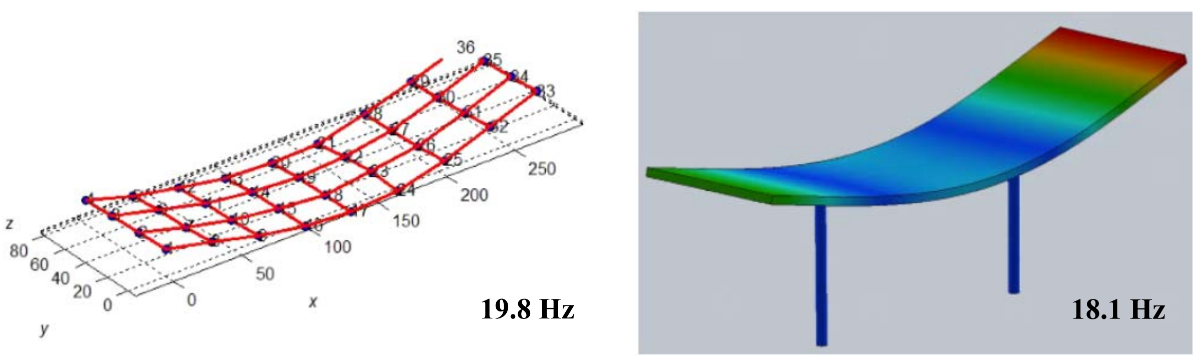
During the last four weeks of the term, the student teams performed an experimental modal test on a structure of their choosing, and they created a finite element model of the same structure. They were instructed to experimentally identify the six lowest nontrivial modes of their structure and compare these with the corresponding modes obtained from their finite element model.

During the last lab period, teams presented their findings and provided explanations for the differences between their experimental and finite element analysis results.

The structures students tested varied in size, shape, and material and had differing boundary conditions. Some structures were brought to the lab to be tested, while others were immovable fixtures elsewhere on campus and were tested on location. For example, a team tested one of the tables in the lab, as shown in Figure 4. The students used 36 impact points in their roving-hammer impact test. The experimental and finite element results for the second mode of the table are shown in Figure 5.

(a) (b)

Figure 4. (a) A table in the lab that a team tested for their final project. (b) The legs are connected to the underside of the table by a ribbed attachment.



(a) (b)

Figure 5. (a) Experimental results and (b) finite element results for the second mode of the lab table in Figure 4.

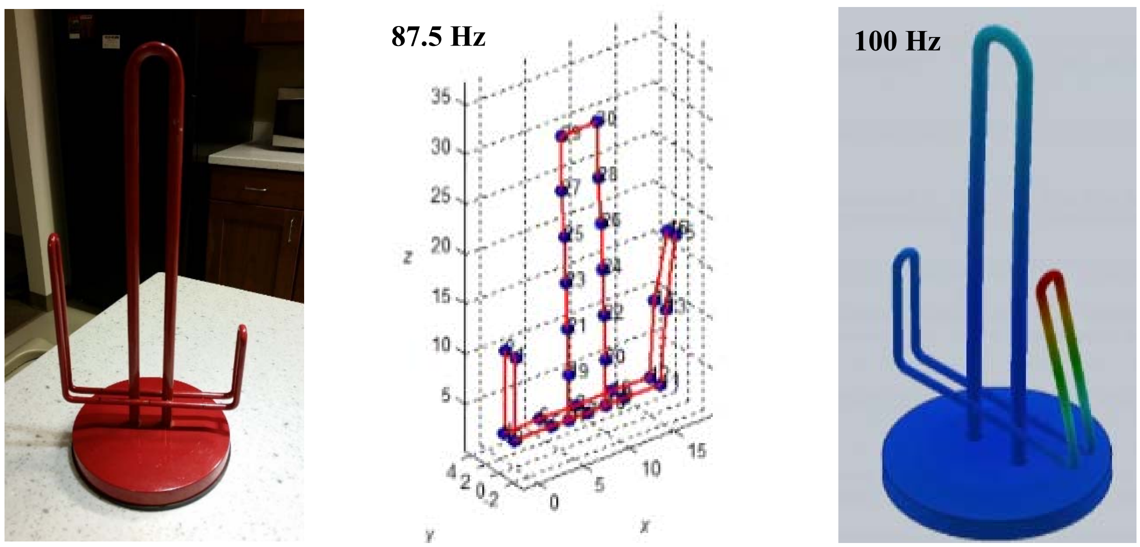
In their presentation, the students provided the following possible explanations for the differences in the natural frequencies they obtained from the experiment and from the model:

- The material properties of the table were unknown.

- The bottom of the support was considered fixed in the finite element model, but in the experiment, it was just sitting on the floor.
- There was a ribbed attachment that connected the legs to the table, as shown in Figure 4(b). In the finite element model, the legs were rigidly attached to the table.

Results for the first mode of a paper towel holder are shown in Figure 6. This is an example of a small structure students brought to the lab to test. The students suspended the object from the ceiling using a bungee cord to simulate free boundary conditions. They provided the following possible explanations for the differences in the natural frequencies:

- The material was unknown.
- The tubes were hollow and had an unknown thickness.
- The base was assumed to be a solid piece of metal.



(a) (b) (c)

Figure 6. (a) A paper towel holder that a team tested for their final project. The experimental and finite element results for the first mode are shown in (b) and (c), respectively.

Assessment

To determine if we achieved our goals, an anonymous survey was distributed at the beginning of the first lab day and at the end of the last lab day. One purpose of the survey was to determine students' self-identified experience using finite element analysis and in performing experimental modal analysis. Students were given a five-level Likert scale with skill levels ranging from "non-existent" to "excellent." The results from asking them to rate their experience in experimental vibrations are shown in Figure 7(a); the results from students' rating their experience using finite element analysis are displayed in Figure 7(b). In these figures, the lowest two levels were combined and so were the top two levels.

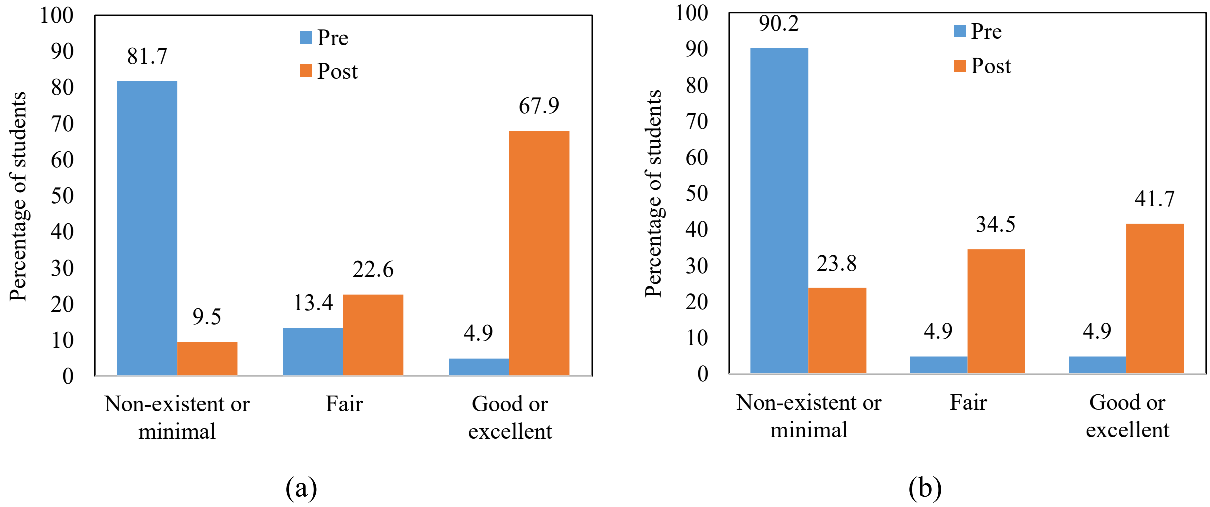


Figure 7. Results of pre- and post-class self-assessment surveys in which students rated (a) their experience in experimental vibrations and (b) their experience using finite element analysis.

From Figure 7(a), we see that prior to the class, over 80% of the students rated their experience in experimental vibrations as non-existent or minimal, and less than 5% rated their experience as good or excellent, which is not surprising. After the class, almost 68% rated their experience as good or excellent. Why almost 10% still rated their experience as non-existent or minimal is concerning and most likely because the project was done in teams. In some teams, it is possible that some members focused on the test while other members focused on the finite element modeling. In Figure 7(b), the percentage of students in the pre-class survey with non-existent or minimal finite element experience was over 90%, and less than 5% rated their experience as good or excellent. At the end of the class, these numbers changed to about 24% and 42%, respectively. Again, the reason some students still rated their experience as non-existent or minimal is most likely due to how students divided the work on the project.

Students were also asked in the surveys to discuss which they would trust more, experimental results or finite element results, and their answers are shown in Table 2. Prior to the class, almost 33% of the students said they would trust finite element results more than experimental results. This was reduced to less than 5% after the class, indicating an increased skepticism of finite element results and an increased trust in experimentation. Students were not given “it depends” as an option, but almost 15% wrote in this answer on the post-class survey.

Table 2. Results from asking students if they trust experimental results or finite element results more.

Survey	Trust experiment more	Trust finite element more	It depends
Pre-class	67.5%	32.5%	0%
Post-class	80.2%	4.9%	14.8%

Finally, students were asked to answer the following question:

> What are some possible explanations for the differences you might see in the results obtained from an experimental modal test and from a finite element modal analysis?

We coded their written results in the pre-class and post-class surveys using the terms listed in Table 3. In this table, we indicate the number of students who used these words in their responses. The increase in the number of students using appropriate technical terms is encouraging. In the pre-class survey, about 11 students mentioned "experimental error" or "human error" to explain the differences. In the post-class survey, only three students mentioned "human error," and it was usually part of a longer discussion. We stressed in this course, as we stress in other classes, that "human error" is not an appropriate explanation for errors in experimental or finite element results.

Table 3. Results from coding students' explanations of differences between test and analysis results.

Code	Number of students	
	Pre-class survey (N=83)	Post-class survey (N=81)
Material properties	15	61
Geometry	0	20
Boundary conditions	0	18
Accelerometers	4	9
Assumptions	4	21

Conclusions

In this study, the lab portion of a Mechanical Vibrations course was evaluated for its effectiveness in improving students' experiences in experimental vibrations and finite element analysis and in improving students' ability to use appropriate technical language when discussing the differences between test and analytical results. As expected, we observed a large number of students—over 70%—reporting good to excellent experience in experimental vibrations by the end of the course and over 40% reporting good to excellent experience in finite element analysis. Our assessment results indicated that most students developed a much more sophisticated understanding of analysis and testing as a result of these experiences, and by the end of the course they were using appropriate terminology when discussing the differences between test and analytical results.

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