# Assessing Critical Thinking in Mechanics in Engineering Education

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Typically, mechanics education in engineering schools focuses on communicating explicit content to students, but deemphasizes the critical thought that underlies the discipline of mechanics. We give examples of the failure of students to apply basic principles of mechanics in solving problems. We develop assessment tools to measure critical thinking in student work, and how well mechanics textbooks engage students in critical analysis. Both tools focus on the treatment of three criteria that we judge to be fundamental, but which are commonly overlooked or undervalued – completeness of free body diagrams, consideration of physical dimension, and careful use of coordinates and sign conventions. Data collected from employing our assessment tools indicates that most of the time, students omit or misunderstand at least one critical idea when solving a problem, even when they obtain a correct answer. We also found that most of the textbooks surveyed had at least one major shortcoming pertaining to our criteria. Mechanics educators should vigorously emphasize fundamental aspects of mechanics, such as those that we suggest here, as a necessary (though insufficient) step to improve the ability of students to think critically and solve problems independently.

## 1. Introduction

**Rooting Mechanics Education in Mechanics.** The science of Mechanics provides the educational foundation for nearly all branches of engineering, due the importance of both (1) its explicit content and subject matter (e.g. the behavior of mechanisms and structures), and (2) its embodiment of analysis and rational thought (e.g. building equations, based on rational models, that describe physical phenomena). Our experience indicates that students and instructors in mechanics courses emphasize the explicit content, but at the expense of developing analytical technique. This view echoes Schafersman, who, though not a mechanician, writes of the need to develop critical thinking in education:

> Perhaps you can now see the problem. All education consists of transmitting to students two different things: (1) the subject matter or discipline content of the course ("what to think"), and (2) the correct way to understand and evaluate this subject matter ("how to think"). We do an excellent job of transmitting the content of our respective academic disciplines, but we often fail to teach students how to think effectively about this subject matter, that is, how to properly understand and evaluate it. [1]

While in the short run the narrower focus on content enables students to (sometimes) get answers to some problems fairly quickly, students often lack even a *basic* working knowledge of how to apply principles of mechanics to approach general problems – even problems that

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require only technique that they have already learned. We proffer that such shortcomings often result from the failure to carefully address fundamentals of mechanics in mechanics pedagogy[4]. Such fundamentals include the completeness of free body diagrams, the consideration of physical dimension, and the careful definition and use of coordinates and sign conventions.

Consider, for example, a student who incorrectly derives the equation of an oscillating mass as $m\ddot{x} - kx = 0$. The sign error in this equation may be, in the student's mind, simply due to a minor algebraic error that is of little consequence – "it's just a sign." But it is likely that the source of this error lies not in careless algebra, but in misunderstanding, or not perceiving, the role of the coordinate $x$ and the need to define it precisely with a sign convention. In this light, the error was arguably *conceptual*. Resolving this problem at its root – not just "fixing" the sign, but really establishing a proper coordinate – would likely lead to a deeper understanding that would be transferable to many other problems.

One approach to correcting this error is to identify its *context*. Fortunately, mechanics naturally lends itself to establishing well-defined categories that may be used to characterize various elements of a given problem. It is well accepted that a given mechanics problem comprises three basic elements: (1) Kinematics (geometrical properties), (2) Laws of Mechanics (balance laws, such as Newton's Laws), and (3) Constitutive Laws (material properties). In their textbook *An Introduction to Statics and Dynamics*, Ruina and Pratrap refer to these elements as the "Three Pillars of Mechanics" [2]. They present the pillars as a fundamental concept in the introductory chapter, and repeatedly refer to them throughout the text[5]. The three pillars constitute a useful, consistent, and philosophically grounded framework with which to formulate and solve essentially all problems. We argue that all students in mechanics should learn to formulate and solve problems according to this framework.

**Critical Thinking.** By *critical thinking*, we mean *a systematic approach to problem solving, including complete and well-conceived problem formulation, generation of a solution, and careful assessment of the solution*. While this definition can be applied across a wide range of disciplines, it is somewhat narrow and operational. Many other definitions abound. Further discussion and ideas may be found in Schafersman [2], Gunnink and Bernhardt [3], Bean [4],

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[4] Perhaps one reason for this is shortcoming in mechanics education is that in typical engineering programs, mechanics is taught as a *service* for degree-bearing disciplines, such as Mechanical Engineering and Civil Engineering. Few universities offer undergraduate degrees in the *discipline* of Mechanics.

[5] In introductory Statics and Dynamics, bodies of interest are often assumed rigid. In such cases, only pillars (1) and (2) are applicable, although pillar (3) is implicitly applied if one views rigidity as a limiting case of constitutive behavior.

Kanaoka [5], and Paul and Elder [6]. Several organizations that maintain related material are the Foundation for Critical Thinking [6], and the Critical Thinking and Pedagogy group at National University of Singapore [7]. Also, The Scientific Reasoning Research Institute is a research organization at the University of Massachusetts-Amherst that has produced literature regarding critical thinking in physics and mathematics education [8].

Employing our definition, *critical thinking in mechanics* refers specifically to critical thinking applied to mechanics problems, using the framework of the Three Pillars. In this sense, was the student who wrote the equation $m\ddot{x} - kx = 0$ thinking critically? We provide a brief commentary below.

> The equation is correctly dimensionally balanced, which indicates at least partially correct application of Newton's 2nd Law, which is included in the 2nd Pillar. On the other hand, as was suggested previously, the sign error may indicate a conceptual error in establishing a coordinate, which is associated with the 1st Pillar. Even if this is the case, perhaps the student was thinking critically in the sense that he or she *applied* the Pillar of Kinematics, but did so erroneously. Finally, the student would be thinking critically if he or she examined the resulting exponential solutions, and realized that these solutions do not represent the expected oscillatory motion. The pinnacle of critical thought would be reached if the student used this realization to re-examine the entire problem solution, identify the error, and re-solve the problem correctly.

## 2. The Breakdown of Critical Thinking in Mechanics Education

In this section we examine evidence from situations in mechanics education in which students fail to employ critical thinking, and in which pedagogical materials fail to engage students in critical thinking. We believe that these examples are representative of typical situations encountered by students and instructors at many institutions, and that they provide a clear and accurate assessment of some fundamental issues that must be addressed.

**Anecdotes from Student Questions.** In the last two years or so, several of our former students have visited us to ask questions pertaining to their current course projects. Strikingly, although their questions varied in topic, all questions fit a disturbing pattern. In each case the students began by saying "we just have one question," implying that only one 'simple' obstacle stood in the way of completing their project. After a few minutes of discussion, and discovering that their question was not so simple, and that it led to new questions, the students would concede that only a week remained in which to complete the final project. Seeing that a week was not nearly enough time to adequately address their new realizations, the students would declare that their initial, though incorrect, assessment of the problem would be "good enough," and that furthermore, they were sure that their instructor did not intend the problem to be as complicated as it now appeared to be.  Below is an example that recounts one specific case, told from the point of view of Prof. Papadopoulos, paraphrased and slightly modified for brevity and simplicity:

> Two former students came to ask for some help with a class project from another instructor's class.  Their project was to analyze the ability of a hook (used by a crane) to raise prefabricated walls upright.  The hook was approximately J-shaped, with a lip.  The hook would grip one end of the wall and lift, while the other end of the wall remained in contact with the ground.
>
> The students said, "We just have one question.  Can we assume that the hook is resisting all of the force?"  I replied, "All of what force?  Your question needs to be more precisely stated."  After pursuing this clarification for a few minutes, I was able to draw out from them that they had really meant to ask, "Can we assume that the hook supports the entire weight of the wall?"  I said, "Draw a free body diagram, and you tell me.  For simplicity, assume that the wall is flat on the ground and is just about to have one end lifted."
>
> After more prodding than should have been necessary for these students, who had completed Strength of Materials (in my class, no less!), one of them drew a simple FBD of the wall, and realized that it could be viewed as a uniform, simply-supported beam.  The hook, therefore, would support *half* the weight of the wall, $W/2$.
>
> We were only just beginning.  I then asked the students to draw a FBD of the hook itself, detailing how it carried the half-weight of the wall.  The students proposed an upward force of the cable, equaling $W/2$, but they had some difficulty in seeing that, in order to balance moments, the force of the wall on the hook was not simply a single downward force.  Rather, I explained, a simple model would be to assume that the wall contacted the hook at two places, on adjacent faces, without friction, rendering the hook a 3-force body, obliquely oriented (see Figure 1).

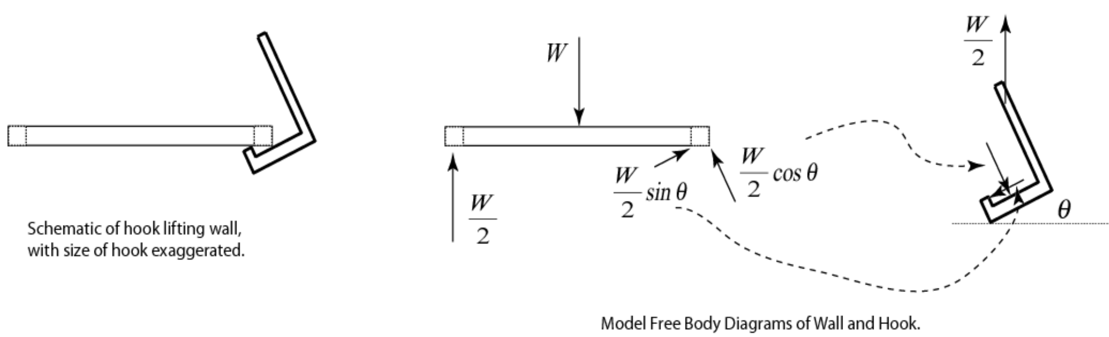


**Figure 1.**  Sketches of hook lifting wall, and suggested Free Body Diagrams.

> Beyond these points, we discussed that the static analysis was only a *prerequisite* to their project. The students readily agreed they needed to analyze the stresses, and suggested that they could apply ideas from Strength of Materials, such as the theories for axial bars and beams.  I commended them for appealing to this line of reasoning, but I cautioned that these approaches were limited, and would be least useful precisely at the locations where the stresses may be most critical.  I mentioned the finite element method, with which neither student had experience, and I also explained more about the importance of properly modeling the boundary conditions.
>
> By this point, we had spent about an hour, and the students appeared somber.  They told me that they thought that my suggestions were correct, but that they didn't have time to try tackling any of them, save for perhaps using the half-weight of the wall, instead of the full weight, in their originally intended analysis.  They told me not to worry, because they were sure that what they

> were already doing would be sufficient to satisfy the expectations of their instructor. I have no idea how they actually solved their problem, and decided 'not to ask, not to tell.'

Regrettably, this example, which is representative of several others, reveals a serious lack of critical thinking on the part of our students. The imperative to address this problem lies well beyond academic perfectionism. The hook project was 'real-world,' and some of its essential analysis was amenable to techniques that the students surely knew. In this case, the static analysis of a simply-supported member was applicable. Had the students been given a simply-supported beam to analyze, they undoubtedly would have analyzed it correctly. But in a context in which the objects at hand were not so literally defined, the students could not apply, from scratch, a simple free body analysis; had they attempted this seriously, they would have at least discovered the answer to their immediate question.

What are the reasons for these lapses in critical thinking, and how can educators address this problem? Complete and definitive answers are likely to prove elusive, as a number of factors – for example, prior mechanics education, innate student ability or interest, demanding schedules and pressures, quality of instruction – are all influential, and are likely to vary significantly from case to case.

Nevertheless, we contend that some definite pedagogical improvements can be advanced, and that while limited in scope, they are necessary if we are to seriously engage our students in critical thinking. In the next two sub-sections we present tools to assess specific aspects of student work and assessments of some textbooks that illuminate some areas where effort should be placed.

**Quantitative Measures of Student Performance.** We identified three specific criteria against which to examine student homework: (1) completeness and correctness of Free Body Diagrams (FBD), (2) incorporating proper physical units (UNITS), and (3) proper use of vectors, coordinates, and sign conventions (VCS). Agreeably, the selection of these criteria is somewhat subjective, but we believe that they are, at a minimum, strong *negative* indicators – that is, students who consistently under-perform on these concepts are likely to have difficulty applying fundamental principles to new problems. They may not be consistent *positive* indicators, as some students will be able to learn how to narrowly satisfy the criteria without true critical engagement. Nevertheless, we contend that emphasizing these criteria in instruction, will, on average, improve students' overall ability in mechanics.

We developed a protocol to evaluate student performance on specific homework problems. For each problem, the instructor or grader would assign a sub-score for each criterion, from 0 – 3:

| | | | | |
|---|---|---|---|---|
| Free Body Diagrams (FBD) | 0 | 1 | 2 | 3 |
| Physical Units (UNITS) | 0 | 1 | 2 | 3 |
| Vectors, Coordinates, Sign Conventions (VCS) | 0 | 1 | 2 | 3 |

where the sub-scores correspond to the following meanings:

0: serious error in final answer, and is attributed to poor application of criterion
1: serious error in final answer, but not attributed to application of criterion
2: final answer essentially correct, despite poor application of criterion
3: final answer essentially correct, and criterion was applied correctly

A more detailed set of rules was established to determine each sub-score (see Appendix A).

The total score $p$ for a given problem is the sum of the three sub-scores, i.e. $p = z_1 + z_2 + z_3$, where $z_i \in \{0, 1, 2, 3\}$. The resulting set of possible scores is {0, 1, 2, 3, 6, 7, 8, 9}. Scores of 4 and 5 are not possible, for if $p \in \{4, 5\}$, there must exist sub-scores $z_i$ and $z_j$ such that simultaneously, $z_i \in \{0, 1\}$, and $z_j \in \{2, 3\}$. However, this cannot occur, because if $z_i \in \{0, 1\}$ the final answer was incorrect; yet if $z_j \in \{2, 3\}$, the final answer was correct. Clearly, these two cases cannot occur simultaneously. Although this dichotomy rewards the attainment of a correct answer, students who obtain correct answers for correct reasons are distinguished from those who get correct answers from incorrect reasons.

The dichotomy of sub-scores between {0, 1} and sub-scores in {2, 3} also implies that each score $p$ arises from a unique triplet of sub-scores, though the ordering of the sub-scores is not unique. For example, a score $p$ = 7 can be realized as (2 + 2 + 3) or (2 + 3 + 2), but 7 cannot be realized as (1 + 3 + 3); in other words, 7 can be realized only from two 2's and one 3. Thus, each score represents a *unique* level of total quality, but a given score does not uniquely indicate the level of quality derived from each individual criterion. As a result, this scoring system provides a *monotonic scale* against which to measure overall quality of work, but does not favor the importance of one criterion over another.

We employed this assessment tool to five different homework problems from Dynamics or Strength of Materials classes. The average scores are reported in Table 1 (N is the number of homework papers evaluated (*incidences*); the total N = 105 arises from five distinct assignments).

| HW | N | FBD | UNITS | VCS | TOTAL |
|---|---|---|---|---|---|
| 1 | 20 | 1.3 | 1.6 | 1.6 | 4.5 |
| 2 | 19 | 1.7 | 1.7 | 1.5 | 4.9 |
| 3 | 29 | 1.9 | 2.3 | 2.1 | 6.3 |
| 4 | 29 | 1.0 | 1.5 | 1.9 | 4.4 |
| 5 | 8 | 2.3 | 2.8 | 2.1 | 7.1 |
| **NET** | **105** | **1.6** | **2.0** | **1.8** | **5.4** |

Table 1. Results of Homework Assessments, by Criterion, and by Homework Assignment.

The results indicate, on average, a modest tendency for students to reach an acceptable final answer (the average score, 5.4, is greater than 4.5). However, the average tendency is also for students to neglect or make a significant error in at least each criterion (each sub-score average is a full point below 3, the score that requires the execution of the criterion without significant error). No single criterion emerges as an area of particular strength or weakness.

We can consolidate the data across all criteria and all assignments to reveal how often a correct (or incorrect) reason correlated to a correct (or incorrect) answer. An explanation for how the data was recompiled is in Appendix A, but roughly speaking, a 'correct reason' correlates to the maximum sub-score for a given criterion. "Immeasurable Reason" refers to incorrect answers that we not directly attributed to any of the three basic criteria (*e.g.* a student who made no sensible progress). Also, the total number of total incidences here is 315, which is 3 times the number (105) of problems studied (recall each problem is scored against 3 distinct criteria).

| N = 3 x 105 = 315 | Correct Answer | Incorrect | Total |
|---|---|---|---|
| **Correct Reason** | 57 (18.1%) | 7 (2.2%) | **64** (21.3%) |
| **Incorrect Reason** | 153 (48.6%) | 29 (9.2%) | **251** (79.7%) |
| **Immeasurable Reason** | | 69 (21.9%) | |
| **Total** | **210** (66.7%) | **105** (33.3%) | **315** |

Table 2. Correlation between Correct Reasons and Correct Answers.

Note that the tabulation generating Table 2 gives a generous impression of student work. For example, a student who arrived at a correct answer on the basis of 2 correct reasons (say FBD and VCS), but one incorrect reason (say UNITS), would contribute 2 tallies for "correct reason, correct answer," and one tally for "incorrect reason, correct answer." (A strict scoring system, requiring that *each* reason be correct for each correct answer, would identify this student entirely in the category of "incorrect reason, correct answer.")  According to this tabulation, most incidences (210, 66.7%), represent correct answers, but also, most incidences (251, 79.7%) also represent incorrect or immeasurable reasoning (immeasurable reasoning likely indicates incorrect reasoning not explicitly measured here, such as trigonometry errors.)

**Analysis of Textbook Materials.** If students are to be challenged to consider precise aspects of mechanics reasoning, such as the criteria we identified, it follows that textbooks have a responsibility to present theory and problems commensurate with this level of detail.  We therefore evaluated several major textbooks, critiquing them on the basis of the same three criteria as with our student assessment: (1) FBD, (2) UNITS, and (3) VCS.  For simplicity, we restricted our evaluations to the standard first three chapters (or equivalent) of introductory Dynamics: Particle Kinematics, Particle Kinetics using Newton's Law, and Particle Kinetics using Energy Methods.  For each category a score was given from 0 – 2, representing our general opinion of the book's presentation of the criteria throughout the three chapters of under examination.  The scores were assigned as follows:

0:  consistently little or improper presentation of criterion
1:  inconsistent presentation of criterion
2:  generally consistent and proper presentation of criterion

In conducting the textbook evaluations, we did not exhaustively catalog every occurrence of each criterion.  Rather, we formed judgements by finding two or three key examples in each text, and then browsing the rest of the selected pages to get a sense of how representative the examples were.  An improved study would more exhaustively track each occurrence, and be evaluated by a panel of several people.  Nevertheless, even if our conclusions are flawed, we submit that we have established a useful protocol for examining textbooks.

| Text | **FBD** | **UNITS** | **VCS** |
|---|---|---|---|
| **Bedford/Fowler [9]** | 2 | 1 | 1 |
| **Beer/Johnston/Clausen [10]** | 1.5 | 1 | 1 |
| **Boresi/Schmidt [11]** | 1 | 0 | 1 |
| **Hibbeler [12]** | 1 | 1 | 2 |
| **Meriam/Kraige [13]** | 1.5 | 0 | 1 |
| **Ruina/Pratrap [2]** | 2 | 2 | 2 |

Table 3.  Textbook Assessment Results.

Table 3 summarizes the results of the textbook evaluations. The texts that were selected were those that were readily available to us. Appendix B contains images of various selections that we examined, with further commentary.

According to our assessment, most textbooks put forth free body diagrams that exclude some forces, particularly in problems concerning energy methods in which some forces may not enter into the calculations (see Appendix B, Figures B1 and B3). *Including all forces is imperative*. Even forces that do no work, or that otherwise may not enter into a calculation, impose real, physical effects, such as enforcing constraints. In some engineering situations, the examination of such forces is crucial. Neglecting any force gives students an exit through which to escape considering and comprehending the true physical reality of the problem at hand, and allows them to pursue lines of thinking according to their own, likely flawed, intuition[6]. Indeed, this is the root cause of why the students who were trying to analyze the J-hook (recounted above) had difficulty in even beginning their problem. The exclusion of a force, even if it appears to be irrelevant, indicates that time and effort is not being committed to cultivating the complete understanding of the problem at hand.

Next, we found that nearly all textbooks frequently exclude units, especially in intermediate calculations, although usually they are attached to the final answer (see Appendix B, Figure B2). Repetitious inclusion of the units, accompanied by emphatic comments, provides a valuable opportunity for the educator to lead the students to realize the power of the units (or more generally, physical dimension) to reveal insights and special properties of the underlying mechanics. Students who develop the habit to consider and include units will be more disposed to critically assessing their own work, and ultimately, they will develop a habit of mind that will assist them in solving problems in more advanced subjects, such as fluid mechanics.

The texts had mixed evaluations on the use of coordinates and sign conventions (see Appendix B, Figures B1 and B3). Most consistently define sign conventions for summing forces and moments. However, the senses for kinematic coordinates are often ambiguously sketched with double-headed or non-headed arrows. A well-defined coordinate should have a single-headed

[6] Physics educators have long perceived the tendency for students to follow their own intuition, rather than the actual dictates of the mechanics. Several researchers have investigated how students' preconceptions interfere with their ability to learn mechanics. Two early works are Clement [14] and McDermott [15]. As this research developed, the Force Concept Inventory (FCI) emerged as a tool to measure students' understandings or misunderstandings of how forces act on bodies (see Hestenes, et. al., [16]). Recently an ASEE group has been formed to collect FCI data (see Gray et. al., [17]). We contend that our recommendation for instructors to unfailingly construct complete Free Body Diagrams will assist students to overcome their incompatible preconceptions.

arrow pointing away from a reference point, defining a positive direction or orientation. This may seem fussy, and agreeably, merely using proper arrowheads without explanation or emphasis is useless. The point is that the careful establishment of a coordinate, including its sense, should be impressed upon the student as a fundamental part of the solution of any problem, and that any careful approach should implicitly include the proper sketching of coordinate directions. The presence of an ambiguously established coordinate is an indication that this discussion of coordinates has not occurred. Such emphasis will hopefully prepare students for more advanced courses, such as Finite Element Analysis, in which a systematic set of coordinate definitions is required to formulate problems and interpret computed results.

## 3. Conclusions and Future Work

We have emphasized the need to train students in mechanics courses to think critically, grounding their problem-solving skills in the core ideas of mechanics itself. We have also defined reasonable and practical measures that can be used to assess both student work and educational materials. Our work shows that in general, students usually miss at least one critical element of a problem, even when they get the correct answer. *It is precisely this gap – between getting the right answer with faulty reasoning, and getting the right answer with correct reasoning – that must be filled if students are to become true problem solvers*. We have also demonstrated that in general, textbooks fall short in demanding critical thinking on the part of their student readers.

We believe that the assessment tools that we present here are useful and innovative, but we also acknowledge their limitations. For example, our homework assessments may not be repeatable. Would other instructors, using our same protocol, give the same analysis of what is acceptably correct? Indeed, the outcomes likely depend on the judgements of the evaluator. However, the assessment tool is likely to be effective for use by a given instructor.

We also recognize that our assessment tool may not categorize student errors uniquely. For example, is an incorrectly labeled force, say $k$ vs. $kx$, an error in the free body diagram, or an error in the use of physical units? Again, different evaluators may make different assessments, and perhaps further work could be done to catalogue and define various kinds of errors. Nevertheless, the overall assessment of how frequently students demonstrate proper reasoning would be reasonably invariant.

Allowing, then, that our assessment tools and procedures are reasonable, we hope that their underlying substance – demanding complete free body diagrams, including physical units, and requiring careful definition and use of coordinate systems and sign conventions – will inform pedagogy in mechanics, so that educators and educational materials will emphasize these concepts. In the future, we hope to use our assessment tools on a larger scale to determine, in fact, whether such pedagogical shifts would really improve students' understanding of mechanics, and their ability to independently solve problems.

Finally, addressing only the specific criteria that we discuss here may not lead to comprehensive improvement in students' critical thinking skills. We have focused on improving analytical technique and developing habits of mind, but perhaps we have missed some other influential concepts. Moreover, in addition to our approach, other teaching strategies, such as writing exercises, assignments with structured iterations for feedback and revision, and design projects, will also help students to cultivate their critical thinking abilities.

In the end, merely presenting the fundamentals of mechanics, even if done correctly, will be useless. It will do little good, for example, to follow even our own suggestions, such as completing free body diagrams or sketching coordinates with single-headed arrows, if not accompanied by a deeper commitment to and insight into student learning. *Our real point is not simply to call for correcting details, but rather, to engender within mechanics pedagogy the well-conceived and planned articulation of the concepts that underpin these details*. Without a serious attempt at this, we will be left with our current situation, so keenly described by Hestenes, et. al.,

> The implications could not be more serious. Since the students have evidently not learned the most basic Newtonian concepts, they must have failed to comprehend most of the material in the course. They have been forced to cope with the subject by rote memorization of isolated fragments and by carrying out meaningless tasks. No wonder so many are repelled! The few who are successful have done so by their own devices, the course and the teacher having supplied only the opportunity and perhaps inspiration. [16]

We educators can more positively influence the learning of our students if we recognize that sound education requires a mutually engaged relationship between the instructor and the student, in which the educator perseveres in challenging the student to understand the subject matter critically, and the student will embrace this challenge as an opportunity for discovery, not as a snare which is to be escaped. If we fail to do our part, as engineering educators, to create this engaged learning environment, we will have made no progress in attaining our goal of training students to think carefully and become independent problem solvers.

# Appendix A.  Evaluation Form for Assessment of Student Homework

**Use of CRITERION [FBD, Units, Coordinates, Vectors & Coordinates]**

| | Flag | Subcore |
|---|---|---|
| CRITERION not used at all | | |
| serious error committed and is due to the absence of the FBD | 1 | 0 |
| serious error committed, but not due to the absence of the FBD | 2 | 1 |
| answer correctly obtained | 3 | 2 |
| CRITERION present but incomplete or incorrect | | |
| serious error committed and is due to the poor FBD | 4 | 0 |
| serious error committed, but not due to the poor FBD | 5 | 1 |
| answer correctly obtained | 6 | 2 |
| CRITERION is present and essentially correct | | |
| serious error committed | 7 | 1 |
| no serious errors and answer correctly obtained | 8 | 3 |

| | |
|---|---|
| correct reason, correct answer | corresponds to Flag #8 |
| incorrect reason, correct answer | corresponds to Flags #3 & #6 |
| correct reason, incorrect answer | corresponds to Flag #7 |
| incorrect reason, incorrect answer | corresponds to Flags #1 & #4 |
| incorrect answer, immeasurable reason | corresponds to Flags #2 & #5 |

# Appendix B.  Excerpts from Textbooks, and Commentary



**15.18** ***Swinging Pendulum***

***Problem Statement*** The pendulum bob shown in Fig. E15.18 weighs $W = 45$ N. It is attached to the end of a light rigid rod whose mass is negligible. The pendulum is rotated from its initial vertical position through the angle $\theta$.

**a.** Determine the potential energy of the pendulum in this position.

**b.** Let the pendulum be rotated until $\theta = 90°$ and then released from rest. Determine the speed of the bob at the instant when $\theta = 0$. Neglect friction of the pin at $A$.

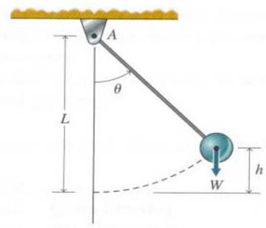


**Figure E15.18**

**Solution** Since friction is negligible, the system is conservative.

**a.** To move the pendulum to the position $\theta$, we must raise the bob from its initial position (taken as the zero configuration, that is, the *configuration of zero potential energy*). Hence, we must raise the bob through a distance $h = L - L\cos\theta$. In doing so, we perform the work

$$U = -U_W = Wh = WL(1 - \cos\theta) \qquad \text{(a)}$$

Equation (a) represents the negative of the work $U_W$ of the gravity force $W$—that is, the work we perform against the force $W$. Therefore, the potential energy of the pendulum at the angle $\theta$ is equal to the work we do in raising the bob:

$$V = U = -U_W = WL(1 - \cos\theta) \qquad \text{(b)}$$

**b.** If the pendulum is rotated through the angle $\theta = 90°$, the bob is raised a distance $L$ and, by Eq. (b), $V = WL$. Therefore, by the law of kinetic energy for conservative systems, we have

$$T_0 + V_0 = T_1 + V_1 \qquad \text{(c)}$$

where the subscript 0 denotes the zero configuration ($\theta = 0$) and the subscript 1 denotes the configuration at $\theta = 90°$. Consequently, $T_0 = \frac{1}{2}mv^2$, $V_0 = 0$, $T_1 = 0$, and $V_1 = WL = mgL$. With these quantities, Eq. (c) yields

$$\tfrac{1}{2}mv^2 + 0 = 0 + mgL$$

or

$$v = \sqrt{2gL}$$

**Figure B1.** In Problem 15-18 from Boresi and Schmidt, the sketch of the pendulum bob is not a true Free Body Diagram, because it does not include the tension. Even though the tension does no work in this model, and thus does not enter into the calculations, the tension force should still be drawn, and the text should explicitly discuss that it is a force that does no work. Including the tension force and these remarks will impress upon the students the importance of considering all reasonable aspects of a problem, and will help them to analyze other problems in which certain forces may not be negligible.

The sketch of the coordinate $\theta$ is drawn correctly, with a single-headed arrow. Since $L$ is a parameter, it is correctly sketched with a double-headed arrow. The variable $h$ should have a single-headed arrow, because it is not a fixed parameter.

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**Example 14.2**

## Connected Objects in Straight-Line Motion

The two crates in Fig. 14.7 are released from rest. Their masses are $m_A = 40$ kg and $m_B = 30$ kg, and the coefficients of friction between crate $A$ and the inclined surface are $\mu_s = 0.2$ and $\mu_k = 0.15$. What is the acceleration of the crates?

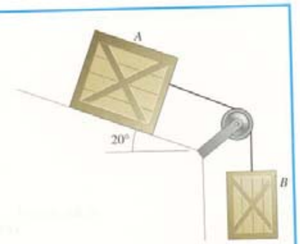



**Figure 14.7**

### Strategy

We must first determine whether $A$ slips. We will assume that the crates remain stationary and see whether the force of friction necessary for equilibrium exceeds the maximum friction force. If slip occurs, we can determine the resulting acceleration by drawing free-body diagrams of the crates and applying Newton's second law to them individually.

### Solution

We draw the free-body diagram of crate $A$ and introduce a coordinate system in Fig. (a). If we assume that the crate does not slip, the following equilibrium equations apply:

$$\Sigma F_x = T + m_A g \sin 20° - f = 0;$$

$$\Sigma F_y = N - m_A g \cos 20° = 0.$$

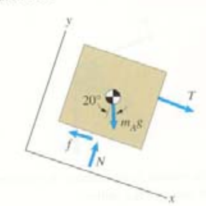
In the first equation, the tension $T$ equals the weight of crate $B$; therefore, the friction force necessary for equilibrium is

$$f = m_B g + m_A g \sin 20° = (30 + 40 \sin 20°)(9.81) = 429 \text{ N}.$$

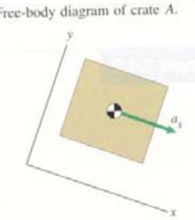
The normal force $N = m_A g \cos 20°$, so the maximum friction force the surface will support is

$$f_{max} = \mu_s N = (0.2)[(40)(9.81)\cos 20°] = 73.7 \text{ N}.$$

Crate $A$ will therefore slip, and the friction force is $f = \mu_k N$. We show the crate's acceleration down the plane in Fig. (b). Its acceleration perpendicular to the plane is zero (i.e., $a_y = 0$). Applying Newton's second law yields




**(a)** Free-body diagram of crate $A$.

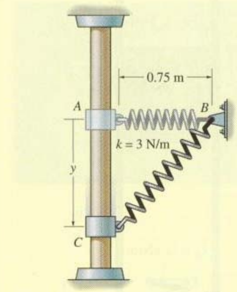



**(b)** The crate's acceleration parallel to the plane.

**Figure B2.** In Problem 14.2, from Bedford and Fowler, the Free Body Diagrams and the basic setup of the equations are correct. However, the physical units are dropped from all terms of the calculations, except for the final answer, in which the units are merely attached. The simplicity of this problem makes it very amenable to including the units.

Beginning students should be trained to include physical units for several reasons. Students can gain practice in manipulating the various units, and thus become familiar with how they must interrelate. Moreover, if students learn to balance units, they can use them to check for errors, such as raising numbers to wrong powers, or forgetting terms in the chain rule. Finally, students who pay attention to physical units will learn more about the fundamental physical nature underlying the problem.

**EXAMPLE 14–11**



(a)

A smooth 2-kg collar $C$, shown in Fig. 14–23$a$, fits loosely on the vertical shaft. If the spring is unstretched when the collar is in the position $A$, determine the speed at which the collar is moving when $y = 1$ m, if ($a$) it is released from rest at $A$, and ($b$) it is released at $A$ with an *upward* velocity $v_A = 2$ m/s.

### Solution

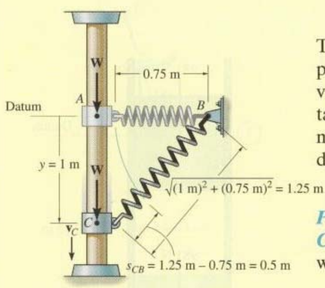
***Part (a)***

***Potential Energy.*** For convenience, the datum is established through $AB$, Fig. 14–23$b$. When the collar is at $C$, the gravitational potential energy is $-(mg)y$, since the collar is *below* the datum, and the elastic potential energy is $\frac{1}{2}ks_{CB}^2$. Here $s_{CB} = 0.5$ m, which represents the *stretch* in the spring as shown in the figure.

***Conservation of Energy***

$$T_A + V_A = T_C + V_C$$
$$0 + 0 = \tfrac{1}{2}mv_C^2 + \{\tfrac{1}{2}ks_{CB}^2 - mgy\}$$
$$0 + 0 = \{\tfrac{1}{2}(2\text{ kg})v_C^2\} + \{\tfrac{1}{2}(3\text{ N/m})(0.5\text{ m})^2 - 2(9.81)\text{ N}(1\text{ m})\}$$
$$v_C = 4.39\text{ m/s}\downarrow \qquad \textit{Ans.}$$

This problem can also be solved by using the equation of motion or the principle of work and energy. Note that in *both* of these methods the variation of the magnitude and direction of the spring force must be taken into account (see Example 13–4). Here, however, the above method of solution is clearly advantageous since the calculations depend *only* on data calculated at the initial and final points of the path.



(b)

**Fig. 14–23**

***Part (b)***

***Conservation of Energy.*** If $v_A = 2$ m/s, using the data in Fig. 14–23$b$, we have

$$T_A + V_A = T_C + V_C$$
$$\tfrac{1}{2}mv_A^2 + 0 = \tfrac{1}{2}mv_C^2 + \{\tfrac{1}{2}ks_{CB}^2 - mgy\}$$
$$\tfrac{1}{2}(2\text{ kg})(2\text{ m/s})^2 + 0 = \tfrac{1}{2}(2\text{ kg})v_C^2 + \{\tfrac{1}{2}(3\text{ N/m})(0.5\text{ m})^2 - 2(9.81)\text{ N}(1\text{ m})\}$$
$$v_C = 4.82\text{ m/s}\downarrow \qquad \textit{Ans.}$$

Note that the kinetic energy of the collar depends only on the *magnitude* of velocity, and therefore it is immaterial if the collar is moving up or down at 2 m/s when released at $A$.

**Figure B3.** In Problem 14-11 from Hibbeler, the first figure is intended to be a descriptive sketch, and as such, is appropriate. But the second figure, intended to have a Free Body Diagram of the collar, omits the tension in the spring. Even though the work done by the spring on the collar can be calculated easily using the spring potential, the spring force should appear in the FBD to emphasize its presence, and impress upon the student that it is enforcing necessary conditions on the motion of the collar.

In the second sketch, a good effort is made to define the stretch of the spring, $s$. The double-headed arrow on $s_{CB}$ is arguably acceptable, if $s_{CB}$ is interpreted as a parameter. However, the stretch varies, and this should be accounted for by a proper coordinate $s$, signified with a single-headed arrow. Then, $s_{CB}$ is the value of $s$ at a particular location.

Also, the energy equation really yields two roots for $v_C$. The reported final answer, downward, is correct, but the reason to select the negative, rather than positive, root is not only omitted, but undermined by the comment that it is 'immaterial if the collar is moving up or down.'

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