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Toward Benchmarking Student Progress in Mechanics: Assessing Learning Cycles through Mastery Learning and Concept Questions

Dr. Christopher Papadopoulos, University of Puerto Rico, Mayaguez Campus

Christopher Papadopoulos is Professor in the Department of Engineering Sciences and Materials at the University of Puerto Rico, Mayagüez Campus (UPRM). He earned B.S. degrees in Civil Engineering and Mathematics from Carnegie Mellon University (1993) and a Ph.D. in Theoretical and Applied Mechanics at Cornell University (1999). Prior to UPRM, Papadopoulos served on the faculty in the Department of Civil engineering and Mechanics at the University of Wisconsin, Milwaukee.

Papadopoulos has diverse research and teaching interests in structural mechanics and bioconstruction (with emphasis in bamboo); appropriate technology; engineering ethics; and mechanics education. He has served as PI of several NSF-sponsored research projects and is co-author of Lying by Approximation: The Truth about Finite Element Analysis. He is active in the Mechanics Division.

Dr. Aidsa I. Santiago-Román, University of Puerto Rico, Mayaguez Campus

Dr. Aidsa I. Santiago-Román is a Professor and Chair in the Engineering Sciences and Materials (CIIM) Department at the University of Puerto Rico, Mayagüez Campus (UPRM). Dr. Santiago earned a BS and MS in Industrial Engineering from UPRM and Ph.D. in Engineering Education from Purdue University. Dr. Santiago has over 20 years of experience in academia and has been successful in obtaining funding and publishing for various research projects. She's also the founder and advisor of the first ASEE student chapter in Puerto Rico.

Her research interests include investigating students' understanding of difficult concepts in engineering sciences, especially for underrepresented populations (Hispanic students). She has studied the effectiveness engineering concept inventories (Statics Concept Inventory - CATS and the Thermal and Transport Concept Inventory - TTCI) for diagnostic assessment and cultural differences among bilingual students. She has also contributed to the training and development of faculty in developing and evaluating various engineering curriculum and courses at UPRM, applying the outcome-based educational framework.

She has also incorporated theories on social cognitive career choices and student attrition mitigation to investigate the effectiveness of institutional interventions in increasing the retention and academic success of talented engineering students from economically disadvantaged families. She's also involved in a project that explores the relationship between the institutional policies at UPRM and faculty and graduate students' motivation to create good relationships between advisors and advisees.

Edward Fritz Hillman, University of Puerto Rico, Mayaguez Campus

Edward Hillman is a recent BSc mechanical engineering graduate from the University of Puerto Rico, Mayagüez Campus. During his Bachelor's he studied the mechanical characterization of full culm bamboo and worked as a systems engineering intern at Lockheed Martin. He plans to pursue a graduate degree in his field of Mechanical Engineering with an interest in Mechatronic system design.

Mr. Gerald Luciano Figueroa, University of Puerto Rico, Mayaguez Campus

Gerald L. Figueroa is an artist and graphics designer, currently expanding his skillset while pursuing a bachelor's degree in Mechanical Engineering. Gerald has always been engaged in diverse projects which involve creating or designing something. While studying at the university, Gerald runs his own small business named ArteFig, in which he converts his digital art into vinyl stickers which he sells both online and in public spaces. Additionally, he works as an Engineering Graphics Developer for Professor Christopher Papadopoulos, where he applies his skills in digital graphic design to generate images for engineering concept questions that students study with. Whether it be for his small business, or for work, or for anything else, Gerald can spend an entire day drawing or designing on his computer. His ultimate goal is to develop his own print and design studio to provide various graphic design, printing, and small-scale manufacturing services to the community.

Isamarie Vega Morales, University of Puerto Rico, Mayaguez Campus

Isamarie Vega is a recent graduate from Mechanical Engineering in the University of Puerto Rico at Mayagüez. Isamarie has worked as professor assistant for around two years for Static Mechanics and Dynamic Mechanics. She has been involved in Education from a young age, learning and helping her mother who is a science teacher, and working as student counselor and tutor for classmates. She is currently pursuing a career in the industry of Engineering Design, with plans of continuing graduate studies after gaining more experience. Additionally, Isamarie owns a 3D printer which she uses as a hobby to learn more and develop design and prototyping skills. Her final goal would be to become an engineering educator, able to teach fundamentals in creative ways that adapt to the new generations of students, including new technologies or dynamics into her lessons.

Toward Benchmarking Student Progress in Mechanics: Assessing Learning Cycles through Mastery Learning and Concept Questions

1. Introduction

This article provides data to benchmark the rate at which student learn Statics concepts based on results of instruction of 107 students in a Statics class at the University of Puerto Rico, Mayagüez (UPRM), taught by the principal author (Papadopoulos), during the Fall 2020 and Spring 2021 semesters. Data is derived from observing learning cycles through which students attempt, re-think, and re-attempt their work. Learning cycles are measured in two ways:

- Through a Mastery Based Learning model, the course is divided into 15 topics (essentially one per week), most of which culminate with a “Mastery Test” that is graded on a pass/conditional/fail basis, with the allowance that students may retake alternative test versions of the same topic multiple times until they pass, or “master”, the topic, corresponding to “A” quality work. Learning cycles therefore occur as students repeat the same topics, and their progress is assessed by passing rates and by comparing evolving responses to the same test topics.

- Concept questions that elicit qualitative responses and written explanations are deployed with each topic. The learning cycle here consists when students respond to a ConcepTest accessed via the Concept Warehouse[1], typically before class, observe the range of answers and comments from other students, receive feedback and/or participate in a peer discussion, and re-attempt the same question. These cycles can occur within a few minutes to over a couple of days, depending on how the ConcepTests are delivered.

Results from both types of cycles imply that the rate of demonstrated learning is quite low in comparison to the time allocated for a typical course. For example, the pass rate for a Mastery Test is typically below 20%, implying that a typical student would require 5 attempts to pass the topic. With ConcepTests, typically fewer than 1/3 of the students who initially answer incorrectly change to the correct answer by the time of the final poll. Yet within these apparently dismal statistics appear promising marks of student progress, but at a rate that is markedly slower than what the academic calendar demands. Because the data is derived from a standard treatment of Statics topics, this work has potential to benchmark rate of student learning, t0 discover of key points of difficulty, and to provide critical reflection on recalibrating expectations and approaches.

2. Learning Cycles

There are various interpretations and uses of “learning cycles” in engineering and science education that are implemented at the scale of classroom activities and lessons within courses[2].

[1] Access the Concept Warehouse at https://newjimi.cce.oregonstate.edu/concept_warehouse/.
[2] Vertical curriculum spirals [16] are also cyclic, but this scale is beyond the scope of this article.

Perhaps most frequently cited is the Experiential Learning Cycle developed by David Kolb, described succinctly in [1] as "a cycle of the Concrete Experience, Reflective Observation, Abstract Conceptualization, and Active Experimentation". The "5E" model developed by the Biological Sciences Curriculum Study (BSCS) consists of stages of Engage, Explore, Explain, Elaborate, and Evaluate. The study's director, Roger Bybee, recommends that "the optimal use of the model is a unit of two to three weeks where each phase is used as the basis for one or more lessons (with the exception of the engage phase, which should be less than a lesson) [with] some cycling of lessons within a phase" [2]. The "Predict-Observe-Explain" sequence [3] [4] might be employed with either method. These approaches are based on constructivist theories of learning and are driven by inductive learning and inquiry.

The idea of a learning cycle employed here is a cycle in which the student engages with a form of instruction, performs a related exercise, receives feedback and/or evaluation, and repeats the exercise or a similar one. This includes, but is not limited to, the examples or theories cited above. For example, learning cycles could be planned and implemented even with deductive lesson styles. Discussion of the relative merits of the forms of pedagogy that best amplify student learning is beyond the scope of this article, but the authors consider themselves to be practitioners of "active learning" strategies. In this article, learning cycles are defined within the use of an overall Mastery Based Learning[3] approach for course delivery, coupled with frequent use of Concept Based Learning using the Concept Warehouse, an online platform that has a bank of thousands of STEM concept questions, called ConcepTests, including nearly 300 for Statics.

2.1 Learning Cycles Based on Mastery Tests

In Mastery Based Learning, also referred to variously as Standards Based Grading [5] [6] or Specifications-Based Grading [7] [8] [9], the course is divided into discrete topics, sometimes called "Mastery Levels", typically ordered by a combination of logical sequencing (e.g., fundamentals followed by applications, elementary followed by advanced, etc.) and prioritization. Each level culminates with an evaluation called a "Mastery Test". In some implementations, students are required to pass, or "master", a given topic before proceeding to the next; in other situations, such as here, students are permitted to progress through all course topics according to a common schedule, but in practice, some might choose not to participate in advanced topics until they achieve the fundamental ones. In either case, Mastery Tests are evaluated on the basis of a "pass" or "fail", leading to a final grade that is based on high quality achievement of discrete competencies, rather than on a smeared average.

In order to promote practice, focus, and motivation, students are allowed to retake multiple test versions for each Mastery Level. A "cycle" is therefore defined in which students can take tests of the same topic multiple times, and progress can be monitored and benchmarked.

2.2 Learning Cycles Based on Concept Questions

Over the last several years, the instructor has integrated "concept based instruction" as part of his overall instructional approach [10] [11]. This consists of deploying, ideally, one or two

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[3] The authors credit their introduction to, and inspiration for, the Mastery Based Learning method to Sara Atwood and Kurt DeGoede at Elizabethtown College [17].

ConcepTests per topic of the course. ConcepTests complement procedural questions in that they typically require no calculations (or very elementary ones), and are intended to elicit higher order reasoning beyond "recipes" and "facts". For example, rather than directly asking a question to, say, determine the force in a given member of a truss, a concept question might be framed as to determine whether the member is in tension, compression, or the zero-force condition. Frequently, students are asked to write written explanations to accompany their answers.

In this class, concept questions are deployed from the bank of "ConcepTests" in the Concept Warehouse. Each question can be answered thoughtfully in as little as 5-10 minutes, but frequently the questions are opened 2-5 days prior to the next class, either as a practice or introductory activity, depending on the situation. Then class discussion can be based on the results and accompanying explanations provided by the students, followed by an opportunity for the students to rethink their answer (sometimes this takes the form of breakout rooms or small groups in which students can explain their reasoning to each other).

3. Course Structure and Management of Learning Cycles

During the Fall 2020 semester, the principal course instructor used the Master Based Learning method for the first time, using his section of Statics with a total enrollment of 49 students (39 male, 10 female). The course was divided into 15 topics or "modules", but some were combined for testing purposes, yielding 13 Mastery Test Levels. In order to earn a given letter grade (A, B, C, D; the institution does not use +/-), the student is required to pass each test corresponding to the grade level, as well as all tests corresponding to the lower grade levels. Approximately one new module is delivered per week, with the first corresponding Mastery Test given during the following week in a timed (30 minute) setting.

3.1 Organization of Modules and Rationale for Benchmarking

As shown in Table 3.1, both the set and arrangement of topics corresponds to what might be found in a "typical" course on Statics. For this reason, the authors propose that benchmarking student progress based on this scheme has meaning for the mechanics education community. Of course, benchmarking requires not only agreement of topics, but also common standards for the style, level, and complexity of the test problems (e.g., to what extent should geometrical and trigonometrical skills be required), as well as common standards for evaluations (e.g., what counts as a minor error vs a substantial error?). Such a uniform approach would require a careful process to be established, as well as widespread, voluntary adoption.

As a step in this direction, this article offers detailed data and account according to the standards established by the lead author. With the interest of multi-institutional collaboration in mind, the Appendix provides a detailed description of each mastery test level, including a Problem Model, statistical summary, and commentary. For now, attention is called to some key features, including: (1) several modules, especially at the D and C levels, require students to draw accompanying vector or force polygons to approximate scale as a means to verify that students understand their calculated results; (2) two different levels focus exclusively on mastering drawing Free Body Diagrams; and (3) the course topics are delivered exclusively in two dimensions until the last level is introduced (the intention is to minimize mathematical complexity and focus on the core 'mechanics' ideas).

Table 3.1. Mastery Levels and Grade Correspondence.

Grade Level	Corresponding Module/Topic	Grade Requirement
D1	Vector Resultants	Must pass all D test levels to earn grade of D
D2 (D2-revised)	Free Body Diagrams/Concurrent Force Systems (Free Body Diagrams/Single Bodies)	
D3 (D4)	Equilibrium/Concurrent Force Systems	
D4 (D5)	Moment Calculations	
D5 (D3-revised)	Free Body Diagrams/Rigid Bodies (Free Body Diagrams/Multiple Connected Bodies)	
D6	Equilibrium/Rigid Bodies	
C1	Trusses/Method of Joints and Zero Force Members	Must pass all D and C test levels to earn grade of C
C2	Trusses/Method of Sections	
C3	Frames and Machines	
B1	Beams/Internal Reactions and Distributed Forces	Must pass all D, C, and B test levels to earn grade of B
B2/B3 Combined test	Area Centroids Area Moment of Inertia	
A1	Friction Modes (Static, Dynamic, Impending)	Must pass all D, C, B, and A test levels to earn grade of A.
A2/A3 Combined test	3D Vector Operations Equilibrium of Rigid Bodies in 3D	

Notes: Each module is designed to be delivered within one week. Topics in (parentheses) represent revisions adopted in Spring 2021. In addition to the Mastery Tests, students were required to complete one team project for the C and D grade levels, and one additional project for the A and B grade levels. For final grading purposes, an allowance was made to credit a passing B1 test as a substitute for C3, as necessary.

3.2 Mastery Test Design, Scheduling, and Grading

Corresponding to the delivery of a new topic in a given week, a new Mastery Test for that topic is deployed in the following week. As a central feature of the Mastery Based method is to allow students multiple opportunities to retake different versions of the same test topic, and anticipating that not all students will immediately pass the tests (as is shown in Section 4, this is the overwhelming majority), a new test version for each prior topic is also deployed each week. This results in the possibility that by the last testing period, a total of 13 tests are deployed, but with a relatively small number of students taking each test. When necessary, students are permitted to choose not to take a test on a given week, for reasons such as illness, internet connection failure, conflict with other exams, or for other reasons that the student determines. Individual make-up tests are not given because the subsequent weeks automatically constitute the opportunity for a make-up.

No explicit limit on the number of retakes was established, but in order to encourage students to focus their attention, as well as to limit the overall grading burden, students were not permitted to take more than three tests per testing period. Therefore, by the time of the fourth testing period, when tests D1, D2, D3, and D4 are all offered, some students need to make a choice. Furthermore, for reasons of test security, each test version is assigned to a unique time slot within the testing period, limiting the possible combinations of testing options. Typically, the tests were scheduled so as to separate recent from past topics, or to deliberately pair some topics to require that students focus their attention on certain fundamentals. For example, a possible schedule for the sixth testing period could be {D6 or D1}, {D5 or D2}, and {D4 or D3}. Optimizing the scheduling procedure is being considered for the coming year. In any scenario, however, students need to be prepared to take responsibility to work within a system that is specified implicitly, not explicitly, which might be uncomfortable for some students.

A team of instructors and tutors grades the tests, with the grading assignments reflective of the schedule. Early in the semester, when only a small number of test topics is deployed, a large number of students takes several of a small number of tests. In this case, it is necessary to divide the grading such that multiple people are grading the same test. Later in the semester, however, when a greater number of topics has been covered, students are spread out such that there are many tests to grade, but with a relatively small number of students per test. In this case, a practice was adopted to divide the grading duties so multiple tests are graded by a single person.

Regarding grading, the terminology of “approved” and “not yet approved”, in lieu of “pass” and “fail”, was adopted, with hope that “not yet” would motivate and encourage students to persist. Early experience showed that expecting near mastery, understood as demonstrating “A” quality work, is not realistic for most students on a timed test. Therefore, to recognize “good” work, understood as “B” quality, an intermediate level of evaluation was developed called “conditionally approved”, in which students are permitted to correct the test in an open “homework style environment” rather than retaking a new test version. As is discussed in Section 4, a substantial number of students depended on the conditional approval as a means to advance, and a surprising number required multiple iterations to complete what were assumed to be straight forward corrections.

An advantage of the Mastery Based method is that grading can be conducted quickly. As is shown in Section 4, a total of 266 Mastery Tests were deployed during the 2020-21 year. This might sound like a preposterous and onerous amount, but during grading, time does not need to be spent breaking down points for partial credit. And while providing useful comments is encouraged, given that students are incentivized to learn from their errors as preparation for a subsequent test, comments can be reduced to highlighting a general area for reconsideration, without full details, resulting in the ability to grade each test within 2-5 minutes. The brevity of comments is particularly used in the case of the conditional approvals; after all, the concession to correct an exam reasonably places the burden of correction on the student, given that the basic expectation is independent mastery. In hindsight, it appears that many students did not routinely perform this type of reflective work, raising the question as to whether the instructor’s job is complete at this point, or whether another mechanism needs to be created and enforced to impose reflective behavior.

Standardization of question design and grading remains an open question. Each test version for a given topic has the same format and expectations, but with a different physical scenario, which leads to an important caveat about uniformity. With Free Body Diagrams, for example, one week the test might include a fixed end and a roller, horizontally or vertically aligned, whereas in another week it might include an inclined slot. Are these comparable? Similarly, in problems that involve equilibrium, varying levels of trigonometrical and geometrical complexity might occur. For example, students have repeated difficulty solving problems with forces whose directions are specified as normal to a given incline, raising the question as to whether it is "fair" to include this kind of situation sometimes but not always. There is no single answer to these questions, but the sense was to allow no more than one significant error corresponding to a prior topic (e.g., using an incorrect slope on a C3 test corresponds to a D1 type error) for conditional approval, and zero such errors for immediate approval. A weekly meeting was held to normalize the grading procedures for each question for that week. The use of a more uniform rubric is being explored for the coming year.

3.3 Further Remarks on Course Delivery

Even before the COVID pandemic, over the last decade, the instructor had used "inverted" or "flipped" classroom teaching, in which content was delivered through animated PPT slide presentations [12], [13] prior to classes, which were designed to be interactive. To better align the content with the new format, and to improve the quality of the student experience during the pandemic, the presentations were overhauled, and a voice narration was added to each of them. As previously noted, a new topic is presented each week. Students were required to asynchronously watch/listen to all presentations for each topic (typically 3-4 presentations of 5-15 minutes each) prior to attending the first corresponding synchronous class meeting (held on Google Meet or Zoom). Additionally, a ConcepTest from the Concept Warehouse was assigned as a companion to each content assignment, but with no credit assigned. Finally, readings and exercises from the assigned textbook (Beer, Johnston, and Mazurek) and Mechanics Map (http://mechanicsmap.psu.edu/) were provided for supplementary content and practice problems, but with no credit assigned. As will be discussed later, the rate of completion of the required, non-credit bearing activities was much lower than in previous semesters in which standard "average-based" grading was used to assign a portion of credit to all activities.

The Mastery Tests were generated through the Moodle LMS, typically using the "Formulas" question type, and were strictly timed to 30 minutes. The Formulas type allows programming questions with randomized input parameters, different for each student, while using a common problem statement and solution formula. This provides both a digital fingerprint for each student's work, as well as an automatic "pre-grading" to see how many students can arrive at the correct answer within the given time frame.

However, there are at least two reasons why reliance on automatic grading is not appropriate. First, automatically graded tests rarely can be designed to award appropriate partial credit reflective of the student's understanding. On one hand, even students who demonstrate the essential reasoning frequently do not attain or enter the correct answer on the platform; conversely, students who attain the correct values might not demonstrate sufficient reasoning. Secondly, the Mastery Tests are designed to test certain concepts that cannot be programmed, such as the drawing of Free Body Diagrams, or drawing a force polygon to graphically verify that the solved forces sum to zero in the vector sense. Therefore, each student was required to submit a written answer within 10 minutes after the closing of the test, to be manually graded by a team of the instructor and assistants (a minimum of 1 assistant per 50 students is recommended). To enforce fairness and improve exam security, tests for which the values submitted on Moodle were absent or inconsistent with the written explanation were subject to disqualification at the discretion of the grader.

Google Classroom was selected as the hub for submitting the written explanations and providing feedback to students, as this platform interfaces with multiple editors and allows for a rapid and continuous communication between the grader and student via a chat that is embedded with the submission. This is particularly useful for students with "conditional approval", so as to allow comparison and continuous chat on successive iterations. Moreover, both students and graders, when given a chance to compare the features of Google Classroom vs. Moodle, preferred Google Classroom for submitting and manual grading of the written tests.

For test administration, it was decided that the most secure, manageable approach was to offer each test version at a common time. Tests were strictly timed to 30 minutes (plus 10 minutes for scanning and uploading written work), using randomized parameters that could serve as a barrier to direct copying, and as a disincentive (exposure via digital fingerprint) to posting in public archives. Tests were "open resource", meaning that students were allowed to consult any reference material of their choosing. However, communication with any other person or service during the test was prohibited. Students were also required to take the test in front of a camera, ideally showing their workspace, but typically students showed only their faces. Flexibility was tolerated here in the interest of respecting practical limitations (e.g., laptop cameras are designed to point at the face, not at the table, and cell phone cameras, which could be pointed at the table, would invariably be used for scanning the final documents). No attempt was made to "lockdown" the browser (e.g., Respondus).

3.4 Remarks on Academic Integrity

An investigation of the 107 total students enrolled for Fall 2020 and Spring 2021 revealed that 6 were guilty of attempting to request a solution of an active Mastery Test from an online service (predominantly Chegg), and 7 were identified to have colluded (on the basis of discovering substantially similar work submitted by multiple students), representing 11% of the base enrollment and resulting in various penalties. Anecdotally, the actual number of violations is believed to be somewhat higher. Moreover, a much higher number of students were verified to have accessed Chegg during the Mastery Tests, in search of prior (inactive) tests on the same topic (this was not penalized). This suggests that students are likely to prioritize "seeking solutions" over "generating solutions" during the test.

Nevertheless, only a small fraction of the penalized cases were "successful" in the sense of leading to a correct answer. This, coupled with the overall passing rate of the tests (frequently below 30%, as discussed in Section 4), suggests that the level of "successful cheating" is low. In fact, recent research [14] has demonstrated that separating students into different testing periods by level of competence drastically reduces incentives and incidence of collusive cheating. Although the Mastery Test schedule employed here is not algorithmically optimized per [14], it has the effect of separating students by level of competence as the semester progresses. As a result, higher performing students, who presumably have lesser incentive to collude, also have relatively few peers with whom to collude; lower performing students, though having both greater incentive and higher numbers of available peers, are unlikely to find reliable co-conspirators. In summary, the Mastery Based testing process established here appears to be robust against successful cheating.

4. Results

In this section, detailed results of each test version, level, and category are provided from both the Fall 2020 and Spring 2021 cohorts. Section 4.1 provides an overall summary at the grade category level, Section 4.2 provides a summary of student performance at the individual test level, Section 4.3 provides further information regarding the history of students through the conditional approval and corrections process, and Section 4.4 provides information regarding student responses to ConcepTests on the Concept Warehouse.

4.1 Grade Level Results

Table 4.1 provides a summary of student performance for the Fall 2020 and Spring 2021 semesters, including initial enrollment, the number of students passing, or "approving", each level, and the number of students leaving unresolved conditional approvals (i.e., did not satisfactorily complete the regrading task).

Table 4.1. Summary of Results per Aggregate Grade Level.

Semester >>	Fall 2020				Spring 2021			
Grade Level Category >>	D	C	B	A	D	C	B	A
Initial Course Enrollment	49	49	49	49	58	58	58	58
Approved Entire Level	30	19	17	10	9	6	2	2
Conditionally Approved/Unresolved	1	3	1	1	15	7	3	0
Approved Entire Level (%)	61.2	38.8	34.7	20.4	15.5	10.3	3.4	3.4
Potential Approval for Level (%)	63.3	44.8	36.7	22.4	41.4	22.4	8.6	3.4
Initial Female Enrollment	10	10	10	10	17	17	17	17
Approved Entire Level	5	4	3	3	0	0	0	0
Conditionally Approved/Unresolved	0	1	0	0	5	1	0	0
Approved Entire Level (%)	50.0	40.0	30.0	30.0	0.0	0.0	0.0	0.0
Potential Approval for Level (%)	50.0	50.0	30.0	30.0	29.4	5.9	0.0	0.0
Initial Male Enrollment	39	39	39	39	41	41	41	41
Approved Entire Level	25	15	14	7	9	6	2	2
Conditionally Approved/Unresolved	1	2	1	1	10	6	3	0
Approved Entire Level (%)	64.1	38.5	35.9	17.9	22.0	14.6	4.9	4.9
Potential Approval for Level (%)	66.7	43.6	38.5	20.5	24.4	14.6	12.2	4.9

Notes: Approved Entire Level = total number of students approving all tests per level. Conditionally Approved/Unresolved = total number of students with at least a conditional approval on all tests per level, but with at least one unresolved correction. (%) are calculated based on total initial enrollment. Potential Approval per Level is based on the total number of students with at least a conditional approval for each test for the level. Data do not reflect actual grade assignments, due to discretional decisions or academic integrity penalties, but in general, unresolved conditional approvals were approved in Spring 2021.

The first striking observation is the sharp contrast in results between the Fall and Spring semester. For the Fall 2020 semester, the pass rate at the D Level, 61.2%, is comparable to recent years' results from traditionally delivered courses, both of the author's sections and for those of other colleagues. The rate of students earning at least C, 38.8%, is somewhat lower than the recent norm of about 50%. This is likely explained by the fact that due to the pandemic, students were extended the option to choose “P/F” instead of a letter grade; it is conjectured that most students with unresolved conditional approvals at the C level would have made the effort to complete the corrections if the P/F option were not available. In addition, a concession was made to award a C for students passing B1 but not C3, further helping to normalize the grades. Interestingly, and encouragingly, the rate of students earning A (20.8%) is higher than in recent semesters under the traditional system. Indeed, several of these students made an exceptional effort and earned the “A” by the last test date. Perhaps the demand of mastery on prior topics also positioned these students well. Nevertheless, it should be noted that the A-level Mastery Tests were simpler compared to what was required in the traditional instruction.

Given that the relative success of the Fall semester occurred during the pandemic, the Spring 2021 semester began with optimism, so it was stunning to see the abysmal results for Spring 2021. Even after promoting “conditional approvals” to full approvals, the final performance lagged far below normal, with 41.4% and 22.4% earning at least D and C, respectively. The potential reasons for this are as follows, and will be carefully considered prior to the Fall 2021 semester:

- The authors appreciate that overall “COVID burnout” is a real factor, and various conversations with students revealed that several were facing a crisis of some sort (family illness or death, personal illness, heavy work schedules, etc.). Nevertheless, the widespread lack of participation demonstrated by students was not anticipated, such as refusal to turn on cameras during synchronous class meetings, or to complete activities exercises that were not credit-bearing; and relatively few attended office hours. Some students even acknowledged that they scheduled other activities, including work, during synchronous class meeting hours. It is unclear why this was not apparent during the Fall 2020 semester.

- Although the instructor used the same methods, and even, in his view, enhanced them – for example, the use of Zoom breakout rooms and a shared whiteboard were systematized – in hindsight, perhaps some changes had negative impacts.

    - In the Fall 2020 semester, the instructor frequently volunteered several extra hours of office hours per week, but in Spring 2021, a stricter schedule was maintained. In anticipation for this, from the first week of the semester, tutors were hired to provide additional tutoring opportunities. Few students made use of these resources, and several students later acknowledged consulting Chegg or privately paid tutors for assistance.

    - Some topics were revised and reorganized. The D1 test was interpreted more strictly to always require the interpretation of an angle greater than 90°, and this proved to be a greater obstacle than in Fall 2020 (even though the same expectation was frequently given). Also, in Fall 2020, the FBD tests were nonconsecutive, and separated as “particle” (D2) and “rigid body”, including reactions (D5); in Spring 2021, these tests were consecutive, and separated as “single body” (D2) and “multiple bodies” (D3), with both tests requiring the recognition of reactions in 2D. The data provided in the Appendix shows that D3 was a particular bottleneck.

It is not yet apparent if these changes are responsible for such drastic change in student performance.

The data in Table 4.1 is also disaggregated by gender. Male and female students appear to have performed comparably to each other in Fall 2020, but the male students performed better than the female students in Spring 2021. Further inquiry is needed to understand this trend.

4.2 Mastery Test Results

The results for each Mastery Level Test topic are provided in Table 4.2. In keeping with the Mastery philosophy, several versions of the same test level were created and deployed. Therefore, the data illustrates the number of versions deployed per test level, the total number of attempts per test, and the corresponding success rates. Note that the aggregate data provided in the rightmost column refer to the success rates based on attempted tests; these are distinct from the data in Table 4.1 where overall success per grade level is reported.

The average approval rate per test – that is, the number of potential approvals divided by the total number of attempts of all versions per test level – is strikingly low. The overall Mastery Test approval rate in Fall 2020 was 26.1%, and in Spring 2021 it was 14.1%. The reciprocals of the approval rates represent the average number of attempts required per student to complete a test level, and this is in the range of 4-7 for the current cohort. However, in practice, the actual average number of attempts per student is lower, because many students do not persist to mastery. In fact, this illustrates a general feature of the Mastery Based Learning method, which is that students have some autonomy to choose the degree to which they complete all topics of the course. According to Kurt DeGoede [15], this can be positive if students choose to focus on mastering the basics:

> Of course, many students must spend additional time on these key topics before proceeding to others, so the increased performance on these skills does come at the expense of other skills. This trade-off is worth the cost when skills are prioritized in tiers or levels requiring mastery of the most important skills before moving into less critical skill sets.

However, the experience of the previous semester suggests that fatigue and other factors were also reasons for students not persisting.

Table 4.2. Summary of Mastery Level Test Results, Fall 2020 and Spring 2021.

Mastery Level Test Topic Fall 2020	Versions Deployed	Total Tests Approved	Total Test Attempts	Test Approval Rate (%)	Attempts on Entire Level	Level Approval Rate (%)
D1 Vector Resultants	11	38	139	27.3	47	80.9
D2 FBDs/Particle	11	34	167	20.4	47	72.3
D3 Equilibrium/Concurrent Force System	15	36	112	32.1	45	80.0
D4 Moment Calculations	12	32	148	21.6	46	69.6
D5 FBDs/Rigid Bodies	12	33	115	28.7	42	78.6
D6 Equilibrium/Rigid Bodies	13	31	143	21.7	40	77.5
D Level Aggregate	**74**	**204**	**824**	**24.8**	**(30 / 38)**	**(78.9)**
C1 Trusses/Joints	11	24	84	28.6	36	66.7
C2 Trusses/Sections	11	25	96	26.0	36	69.4
C3 Frames & Machines	10	19	103	18.4	33	57.6
C Level Aggregate	**32**	**68**	**283**	**24.0**	**(19 / 33)**	**(57.6)**
B1 Beams/Distributed Loads	9	19	56	33.9	26	73.1
B2/B3 Centroids/Moment of Inertia	7	17	41	41.5	25	68.0
B Level Aggregate	**16**	**36**	**97**	**37.1**	**(17 / 23)**	**(73.9)**
A1 Friction Laws	6	11	30	36.7	17	64.7
A2/A3 3D Vector Operations/Equilibrium	3	11	28	39.3	15	73.3
A Level Aggregate	**9**	**22**	**58**	**37.9**	**(10 / 15)**	**(66.7)**
Course Aggregate	**131**	**330**	**1262**	**26.1**	**(30 / 49)**	**(61.2)**
Mastery Level Test Topic Spring 2021	**Versions Deployed**	**Total Tests Approved**	**Total Test Attempts**	**Test Approval Rate (%)**	**Attempts on Entire Level**	**Level Approval Rate (%)**
D1 Vector Resultants	18	31	209	14.8	56	55.4
D2 FBDs/Single Bodies	16	37	207	17.9	56	66.1
D3 FBDs/Multiple Bodies	15	22	253	8.7	53	41.5
D4 Equilibrium/Concurrent Force System	14	19	156	12.2	46	41.3
D5 Moment Calculations	13	19	115	14.1	43	44.2
D6 Equilibrium/Rigid Bodies	12	16	102	15.7	42	38.1
D Level Aggregate	**88**	**144**	**1042**	**13.8**	**(9 / 41)**	**(22.0)**
C1 Trusses/Joints	10	9	72	12.5	29	31.0
C2 Trusses/Sections	9	10	60	16.7	26	38.5
C3 Frames & Machines	9	9	58	15.5	17	52.9
C Level Aggregate	**28**	**28**	**190**	**14.7**	**(6 / 16)**	**(37.5)**
B1 Beams/Distributed Loads	7	4	26	15.4	11	36.4
B2/B3 Centroids/Moment of Inertia	5	5	23	21.7	10	50.0
B Level Aggregate	**12**	**9**	**49**	**18.4**	**(2 / 9)**	**(22.2)**
A1 Friction Laws	4	2	23	8.7	11	18.2
A2/A3 3D Vector Operations/Equilibrium	3	2	7	28.6	6	33.3
A Level Aggregate	**7**	**4**	**30**	**13.3**	**(2 / 6)**	**(33.3)**
Course Aggregate	**135**	**185**	**1311**	**14.1**	**(9 / 58)**	**(15.5)**

Notes: The number of approvals is the total of all test approvals per category. The Test Approval Rate is the number of approvals divided by the number of attempts corresponding to each test level. Attempts on Entire Level is the total number of students with at least one test per level. Numbers in (parentheses) represent total numbers of students (passing / attempting) an entire category.

4.3 Conditional Approval and Correction Process

As was noted previously, an intermediate level of evaluation, called "conditional approval", was used to enable students who perform "B" quality work on the timed Mastery test to earn full approval through a "correction and regrade" process, in lieu of retaking another version of the same test. Students might earn a conditional approval for any of the following: making no more than one "moderate" error related to a prior topic, as long as the logic of the core topic is correct; incorrectly answering an interpretative question that is considered beyond the 'minimal' competency'; or some combination of these at the discretion of the grader. In grading, minimal explicit detail was provided, such as, "there is an error here, can you see it?", in order to give the student the opportunity to rethink the problem. Nevertheless, the working assumption was that students with a conditional approval were "close", and therefore would be able to make corrections relatively easily.

In practice, the process for correcting and regrading the conditionally approved tests proved interesting and not as 'automatic' as had been expected. Frequently, students required more than one iteration of corrections before reaching approval (some of which concluded with the instructor offering an approval for borderline quality work). Table 4.3 provides statistics regarding the history of the conditional approval corrections process from Fall 2020 (similar data for Spring 2021 have not yet been tabulated).

Table 4.3. Tabulation of Iterations for Correcting Conditionally Approved Tests.

Grade Category	Test Level	Number of Students by Number of Iterations Required: 1 Iteration	2 Iterations	3 Iterations	4 Iterations	5 or more Iterations	Total Conditionally Approved Cases
D	D1	12	3	2	0	0	17
	D2	18	4	2	1	1	26
	D3	10	3	3	0	1	17
	D4	8	5	2	2	0	17
	D5	7	5	2	0	0	14
	D6	9	8	4	0	0	21
	D Total	**64**	**28**	**15**	**3**	**2**	**112**
C	C1	14	6	0	1	1	22
	C2	9	5	1	0	0	15
	C3	10	2	2	0	0	14
	C Total	**33**	**13**	**3**	**1**	**1**	**51**
B	B1	13	1	0	0	0	14
	B2/B3	11	1	0	0	0	12
	B Total	**24**	**2**	**0**	**0**	**0**	**26**
A	A1	6	1	1	0	0	8
	A2/A3	6	0	0	0	0	6
	A Total	**12**	**1**	**1**	**0**	**0**	**14**
	Course Total	**133**	**44**	**19**	**4**	**3**	**203**

Notes: There are slight discrepancies in the total tallies in the rightmost column compared with the data in the rightmost column of the tables in the Appendix. This is due to minor differences of interpretation or updates.

According to Table 4.3, a total of 203 Mastery test attempts were initially assigned "conditional approval", representing 16.1% of the 1262 total attempts of all tests (see Table 4.2). It is suspected that this proportion is moderately higher for the Fall 2021 cohort. As can be seen, in most cases (133/203, or 65.5%), students satisfactorily completed the corrections the first time, but a significant number required repeat attempts.

There are a variety of reasons for this. First, in some cases, the designation of "conditional approval" was perhaps an overestimation of the student's understanding at the time of that particular test. However, in such cases when students take the correction process seriously, the method leads to growth in understanding; it is not a "free pass".

On the other hand, there is evidence to suggest that students will interpret the regrading task as to address only what is explicitly marked, and not as a holistic exercise to examine everything for consistency and high mastery. Figure 4.3.1 provides an example of a series of corrections for a D1 level test in which the task is to calculate the vector resultant of three vectors, and then sketch the corresponding vector polygon to scale. Even though the numerical calculations were corrected after the first iteration, the corresponding vector polygon took two additional iterations for completion. The process exposes the possibility that the student was possibly not "expecting" to update the force polygon accordingly, and then showed a lack of immediate insight on how to make the correspondence. It is an open question whether this represents "thoughtful misunderstanding" or a lack of commitment to critical thinking.

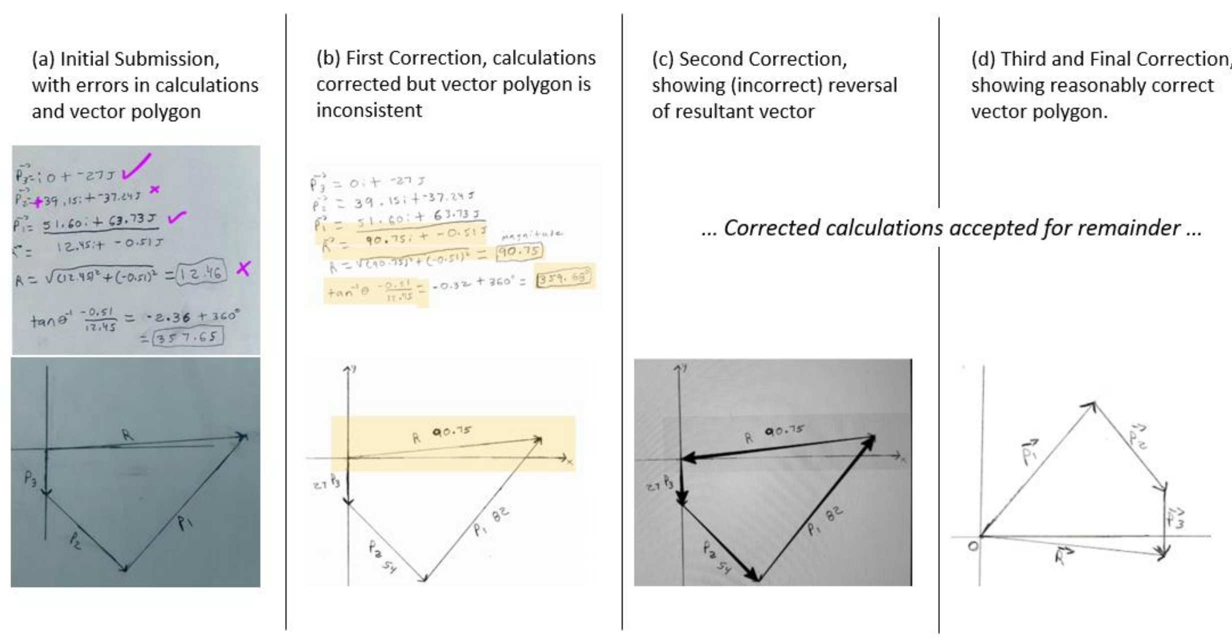

Figure 4.3.1. Progression of Corrections for a Conditionally Approved D1 Test.

(a) In this D1 test (excerpted) a sign error was committed, leading to an incorrect result. (b) The first correction now has the correct calculations, but the vector polygon is still incorrect (the resultant should point downward). (c) A quick “correction” was made, reversing the entire arrow, but not realizing that the “head to tail” addition rule is now violated. (d) A qualitatively correct vector polygon, approximately to scale, has now been presented.

Another example is shown in Figure 4.3.2. Here the task is to draw the Free Body Diagram of a system and various subsystems, showing consistent labeling according to Newton’s Third Law, otherwise referred to as the Principle of Action-Reaction. Here, the initially submitted response had an error of omission – a normal force was missing (details are provided in the caption of Figure 4.3.2). As suggested by the number and type of iterations here, the student is attempting a series of edits that, while possibly representing genuine misunderstanding, suggest that the student did not take time to carefully consider the Principle of Action-Reaction. As a side comment, this example, as well as many others, illustrates that this “well-known” principle is not well understood.

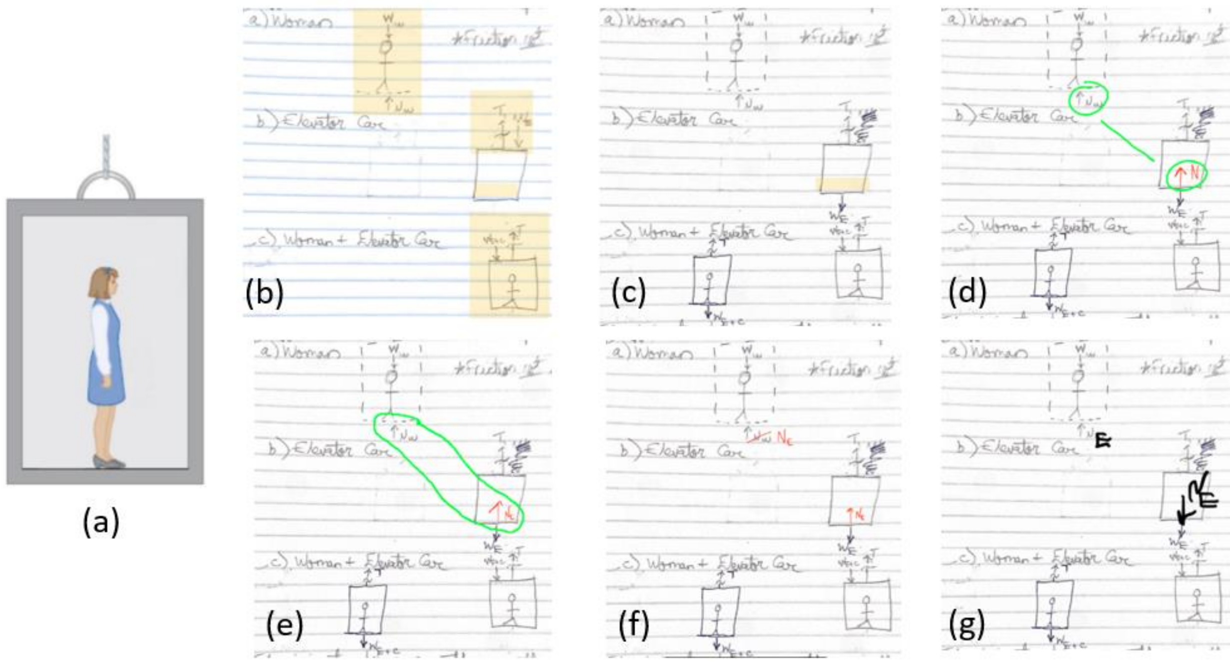

Figure 4.3.2. Progression of Corrections for a Conditionally Approved D2 Test.

(a) The student is presented with a diagram of a woman standing in an elevator, and is asked to draw consistent Free Body Diagrams of the elevator car, the woman, and the combined woman + elevator car system. (b) The initial response of the student. The orange blocks correspond to comments from the grader. Note that the work is all correct except that the normal force of the woman on the elevator floor is missing, and this is called out by a comment box. (c)-(f) The student undertakes various iterations, each time not quite generating a consistent set of diagrams. (g) The student reaches a correct response. Not shown is an extensive chat in the Google Classroom platform in which the instructor gives continued advice, but without explicitly stating what the answer should be.

4.4. ConcepTest Results

As a preamble, note that since the Mastery Based Learning method was adopted, there has so far been no provision for partial or bonus credit. Therefore, unlike in previous semesters, relatively few students completed the ConcepTests assigned before class. This led to some class time being used for completing this activity, delaying and undermining the overall process of discussion, feedback, and re-attempt.

Of the several ConcepTests deployed in Fall 2020, three had sufficient data to compare initial vs. re-poll results. In each case, the repoll was conducted after a focused class discussion with hints, such as "draw a Free Body Diagram" to help you to see what is happening. These results are summarized in Table 4.3.

As indicated by the table, the number of students participating in the "learning cycle", that is, those who submit both an initial and repoll response, is relatively small, consistently below half of the class. Not surprisingly, the participation decreases as the level increases. Secondly, this data gives evidence that short-cycle processes, such as in-class discussion and feedback, is not sufficient to change many minds. Students tend to remain with their initial answer. Fortunately, the small number of students who correct their responses is greater than the even smaller number of students who migrated away from a correct to an incorrect response. It is not immediately clear what could be improved in the class discussion dynamic to improve this overall pattern. This data is also consistent with previously reported data [10].

Table 4.3. Results of Initial vs. Repoll of Concept Questions (Fall 2020)

Mastery Topic Concept Warehouse ID	**A1 Friction ID 4497**	**C3 Frames and Machines ID 5134**	**C1 Trusses/Joints ID 6002**
Incorrect → Incorrect	5	9	3
Correct → Incorrect	0	2	2
Incorrect → Correct	3	3	6
Correct → Correct	2	6	6
Subtotal	**10**	**20**	**17**
Single Attempt Correct	4	4	5
Single Attempt Incorrect	9	4	11
Total	**23**	**28**	**33**
Base Enrollment	**49**	**49**	**49**

Notes: Results are classified by response type. For example, "Incorrect →Correct" indicates that the student initially answered incorrectly, and then answered correctly, etc. Questions can be queried (searched by ID number) from the ConcepTests menu at the Concept Warehouse: https://newjimi.cce.oregonstate.edu/concept_warehouse/.

As a final comment, there is a positive correlation between net Mastery Test performance (total approvals vs total attempts) and net ConcepTest performance (total correct responses vs total attempts, with initial polls and repolls counted equally), with the Pearson correlation coefficient of about 0.54 (N = 39, $p < 0.0004$, 2-tailed). This is analysis is unweighted in the sense that it did not take into account the number of attempts, only average performance. Figure 4.4 shows the corresponding scatter plot. This is not a statement of causality, but it is reasonable to hypothesize that students who exhibit at least one of aptitude and dedication (willingness to push go through the respective cycles) will perform well in both environments.

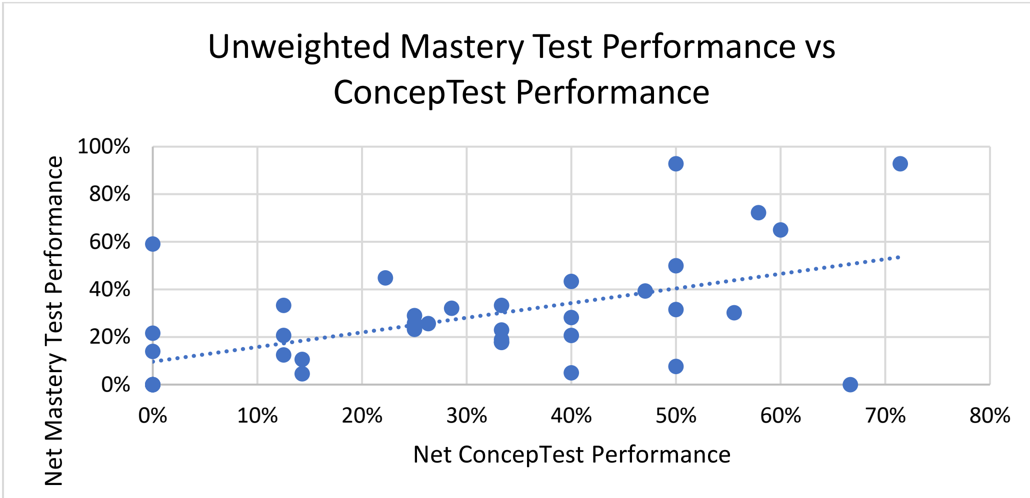

Figure 4.4. Mastery Test Performance vs. ConcepTest Performance.

5. Discussion

On the surface, these results raise significant concerns. The data demonstrate that typically, students require much more time than is allocated to achieve mastery and understanding of concepts, whether on the scale of in-class concept exercises or weekly mastery learning and testing cycles. This should provoke reflection at the level of individual instructors, colleagues within departments who teach mechanics, and in bodies such as the ASEE Mechanics Division, about what are reasonable expectations for student progress, and what might be measures for improvement. Although the data presented here is insufficient to recommend any sweeping changes, due to the correspondence of the topics with standard treatments of Statics, the possibility for replication exists, which could then lead to a larger database and set of experiences. At a minimum, there is a potential to develop standardized competencies that could be interpreted relatively uniformly across institutions.

On the related topic of assessment, an appeal of the Mastery Based Learning method is that it can be aligned with assessment for accreditation and data can be more easily disaggregated. For example, consider a typical outcome for a Statics class, such as “the ability to draw a Free Body Diagram? In what context does this occur, and at what level of complexity? Since the Free Body Diagram is ubiquitous, the objective becomes difficult to assess without detailed parsing of partial steps of complex problems. Mastery Based Learning allows for the possibility of an agreed-upon minimum standard, such as presented here in Levels D2 and D5. Higher level topics that include Free Body Diagrams can therefore be assessed primarily on the merits of their focus, and if there is a Free Body Diagram error, a simple notation such as “FBD error” can quickly be made.

Still, the principal instructor confesses a concern when deliberating whether or not to try the Mastery Based Learning approach, which was that the Mastery Tests would be too well-structured and predictable, leading students to pseudo-memorize how to do the problems and pass the tests without merit. However, the data of the overall approval rate per test – in the range of 15%-30% – proves that this is not the case. In fact, as a humorous anecdote, the when the instructor gave the first Mastery Test in Fall 2020 (D1 v1), by mistake, he issued the exact same problem that was distributed for practice. But even with this exposure, only 31.1% of students approved the test! This also suggests that even though the tests were virtual (with human proctoring by video), there appears to be no widespread successful cheating, although a recent investigation led to several problems being "unposted" from Chegg.

Nevertheless, there remains a desire to create richer, ill-structured problems, beyond what can be achieved in a Mastery Test. To this end, the instructor introduced projects that included elementary sensitivity analysis, design, and consideration of social, ethical, and global issues.

This initial work suggests several avenues for future research. First, in the overall sense of student rate of successful performance, the data by themselves do not indicate underlying reasons. Is the 15%-30% approval rate on the Mastery Tests due to lack of effort, evidence of poor preparation, or genuine difficulty for the average engineering student? The authors believe that focused interviews could be designed to probe these questions.

A related line of inquiry that could be explored through interviews is to what extent, if any, does the Mastery Based Learning method change students' habits? It has been reported in the literature that the method encourages students to focus more on their learning, and less on their grade (due in part to the lower-stakes nature of the weekly tests). The author has seen an overwhelming attendance at office hours, as was expected, although preliminary investigation suggests the students in the Mastery cohort benefit more from office hours than students in traditional settings. This needs to be explored further.

How do students who complete a Mastery Based course fare in subsequent courses? Does a student with a D have sufficient preparation to continue to a future course, particularly if they did not attempt the higher level topics?

There are also research questions at the level of each test. As part of grading, the authors have compiled notes which suggest codes for both positive and negative student responses. Each Mastery Level Test constitutes a trove of mixed quantitative and qualitative data to be analyzed. To give an example, with the singular focus on Free Body Diagrams with the D2 and D5 tests in Fall 2020, the authors detected that students had difficulty identifying the points of contact on a body as a precursor to drawing the applied loads or reactions. This led to a change in philosophy of teaching Free Body Diagrams to give an explicit instruction to identify contact points and write other verbal descriptions of how forces and torques are transmitted, which was employed in Spring 2021. Although the approval rates for the new version was only 8.7%, the authors believe that students became better at isolating bodies and identifying contact points. What remains is to determine how to better understand reaction types. A future article will address this in detail.

6. Conclusions

The data presented here provides a window to benchmarking the rate of student progress in Statics. The corresponding course delivery method, Mastery Based Learning, provides a division of the Statics topics into discrete competencies that can be standardized and used across institutions. The results here suggest that students require more time and more iterations to master even elementary topics than is typically allocated. It is recommended for instructors to reflect on this and consider how they might recalibrate their expectations of total student performance, but with the shift that a minimum level of performance represents genuine mastery of fundamental principles and topics. Additionally, the contrasting results in performance from Fall 2020 to Spring 2021 suggest that the COVID pandemic has significantly affected student learning in a negative manner.

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Appendix. Mastery Test Level Details.

The Appendix contains detailed descriptions, data, and commentary for each Mastery Test level, including:

- A Problem Model adapted from a publicly distributed practice version, slightly edited for publication;
- Statistics about attempts and approval rate for each test;
- A commentary discussing observations and impressions of the authors as a result of teaching and grading.

Note that the topics D3-D5 from Fall 2020 were reorganized for Spring 2021. Topics D3, D4, and D5 from S2021 correspond to topics D5, D3, and D4, respectively, from F2020. Therefore, the data tables for these corresponding topics are paired. Additionally, the D2 topic was revised. These correspondences are indicated with (*).

D1 Level Mastery Test: Vector Resultants

Problem Model.

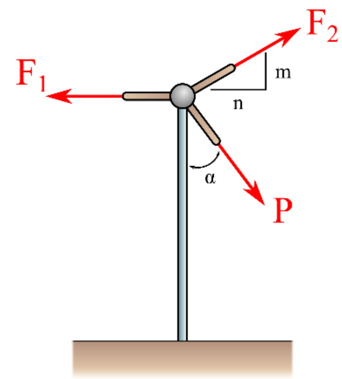
Consider the given set of vectors with the following parameter values: $F_1 = 6.4$, $F_2 = 6.6$, $m = 9.8$, $n = 2.5$, $P = 9.5$, and $\alpha = 244^o$.

1. Determine the magnitude of the resultant of the vectors.

2. Determine the angle of the resultant vector, measured in degrees, CCW from the positive *x*-axis. Your answer should be positive. For example, -30^o should be expressed as 330^o.

3. Sketch the "vector polygon" to approximate scale, showing the initial vectors adding to the resultant.

Figure not necessarily drawn to scale relative to your given data.



Results.

D1 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	Total
Attempts	**45**	**29**	**23**	**9**	**10**	**6**	**3**	**5**	**5**	**3**	**1**	**139**
Immediate Approval	11	4	3	1	0	1	0	1	0	0	0	21
Conditional Approval	3	2	4	0	0	2	0	1	3	1	1	15
Total Approvals	**14**	**6**	**7**	**1**	**0**	**3**	**0**	**2**	**3**	**1**	**1**	**38**
Pending Approval	1	0	0	0	4	0	2	0	0	2	0	9
Approval Rate (%)	**31.1**	**20.7**	**30.4**	**11.1**	**0.0**	**50.0**	**0.0**	**40.0**	**60.0**	**33.3**	**100.0**	**27.3**
Immediate Rate (%)	24.4	13.8	13.0	11.1	0.0	16.7	0.0	20.0	0.0	0.0	0.0	15.1
Potential Rate (%)	33.3	20.7	30.4	11.1	40.0	50.0	66.7	40.0	60.0	100.0	100.0	33.8

D1 Spring 2021	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	v13	v14	v15	v16	Total
Attempts	**51**	**40**	**31**	**15**	**20**	**14**	**8**	**2**	**5**	**3**	**6**	**3**	**4**	**3**	**3**	**1**	**209**
Total Approvals	**9**	**4**	**6**	**2**	**3**	**1**	**2**	**0**	**0**	**1**	**0**	**2**	**0**	**0**	**1**	**0**	**31**
Pending Approval	2	3	4	0	3	2	2	0	0	1	0	0	2	1	0	0	20
Approval Rate (%)	**17.6**	**10.0**	**19.4**	**13.3**	**15.0**	**7.1**	**25.0**	**0.0**	**0.0**	**33.3**	**0.0**	**66.7**	**0.0**	**0.0**	**33.3**	**0.0**	**14.8**
Potential Rate (%)	21.6	17.5	32.3	13.3	30.0	21.4	50.0	0.0	0.0	66.7	0.0	66.7	50.0	33.3	33.3	0.0	24.4

Commentary. Students do demonstrate mastery of the basic skills to add force components and compute magnitudes of vectors. But this "straightforward" exposes other weaknesses. Few students will of their own instinct draw their figure to the scale and proportion corresponding to their data, and they often submit answers with calculations inconsistent with their figures. This foreshadows an entire pattern of difficulty interpreting visual data and sign conventions over the remainder of the course. Other details that confound students:

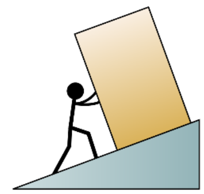
- The angle is often given in a "nonstandard" manner, requiring interpretation of an angle greater than 90^o, and from a reference axis different than $+x$.
- The requirements to draw the vector polygon and interpret the final angle relative to the standard form are atypical, and students appear to deprioritize their importance.
- Students have difficulty expressing the angle or slope of a line that is perpendicular to a given line or direction.

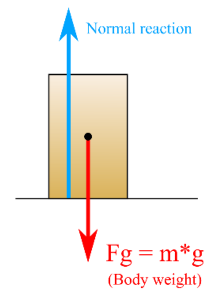
D2 Level Mastery Test: Free Body Diagrams of Concurrent Force Systems*

Problem Model.

1. Consider the system in which someone is pushing a large box up an inclined ramp which has a *rough* surface. Draw the Free Body Diagram of (a) the person, (b) the box, and (c) the combined system of person + box. Be sure that your diagrams are labelled consistently.

2. Consider a possible Free Body Diagram of a box sitting on a smooth floor. Is the FBD drawn appropriately? Justify your answer.





Results.

D2 Spring 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	Total
Attempts	**39**	**33**	**22**	**22**	**11**	**9**	**9**	**7**	**8**	**6**	**1**	**167**
Immediate Approval	3	3	3	8	2	2	2	1	3	4	0	31
Conditional Approval	1	2	0	0	0	0	0	0	0	0	0	3
Total Approvals	**4**	**5**	**3**	**8**	**2**	**2**	**2**	**1**	**3**	**4**	**0**	**34**
Pending Approval	0	0	1	2	1	0	0	2	0	2	0	8
Approval Rate (%)	**10.3**	**15.2**	**13.6**	**36.4**	**18.2**	**22.2**	**22.2**	**14.3**	**37.5**	**66.7**	**0.0**	**20.4**
Immediate Rate (%)	7.7	9.1	13.6	36.4	18.2	22.2	22.2	14.3	37.5	66.7	0.0	18.6
Potential Rate (%)	10.3	15.2	18.2	45.5	27.3	22.2	22.2	42.9	37.5	100.0	0.0	25.1

D2 Spring 2021	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	v13	v14	v15	Total
Attempts	**51**	**44**	**38**	**21**	**15**	**6**	**8**	**3**	**8**	**4**	**5**	**2**	**1**	**0**	**1**	**207**
Total Approvals	**3**	**5**	**11**	**8**	**1**	**2**	**0**	**1**	**2**	**1**	**1**	**1**	**0**	**0**	**1**	**37**
Pending Approval	1	1	3	0	2	0	2	0	1	1	2	0	0	0	0	13
Approval Rate (%)	**5.9**	**11.4**	**28.9**	**38.1**	**6.7**	**33.3**	**0.0**	**33.3**	**25.0**	**25.0**	**20.0**	**50.0**	**0.0**	**--**	**100.0**	**17.9**
Potential Rate (%)	7.8	13.6	36.8	38.1	20.0	33.3	25.0	33.3	37.5	50.0	60.0	50.0	0.0	--	100.0	24.1

Commentary. Free Body Diagrams are at the essence of mechanical reasoning, but even simple situations familiar in everyday present a level of abstraction (including making reasonable assumptions) that prove challenging. Ideas such as action-reaction, internal vs. external, and direct vs. indirect are clearly not well understood upon entry to the course learning these "rules" and conventions is slow. Errors reveal confusion between weight and "associated" normal contact forces, and foreshadow the misconception that certain "associated" forces directly balance each other vs. the mutual balance encoded in the equations of equilibrium.

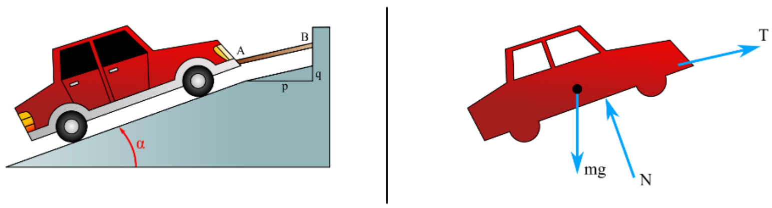
*The D2 problem category was redefined for S2021 to focus on the drawing of a single rigid body, with descriptions of what entity causes each force. This proved difficult. Students tend to say things like "the weight of the car causes the normal force on the wheels", and not "the ground causes the normal force on the wheels".

D3 Level Mastery Test: Equilibrium of Concurrent Force Systems*

Problem Model.

A car has mass m = 1615 kg and rests on a ramp inclined at angle α = 11.8º. It is also supported by a cable inclined with slope given by p = 10.1 and q = 1. The car's brakes are not engaged and therefore the wheels provide no frictional reactions. A Free Body Diagram is given for the car in which N represents the total equivalent normal force of the ramp acting on the car.

1. Using the given Free Body Diagram, write the Equations of Equilibrium that would allow you to solve for the reactions N and T. Your equations should be rearranged such that the unknown terms on the left, and the known terms on the right.
2. After writing your Equations, solve the equations (you are allowed to use a pre-programmed solver).
3. After determining the solution, draw the final force polygon of all forces, to approximate scale, showing that the resultant of the force vectors is zero.



Results.

D3 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	v13	v14	v15	Total
Attempts	**39**	**10**	**8**	**5**	**5**	**4**	**8**	**10**	**7**	**3**	**2**	**3**	**3**	**3**	**2**	**112**
Immediate Approval	13	0	2	0	0	2	1	0	0	0	0	0	0	1	0	19
Conditional Approval	5	0	1	2	1	0	0	1	2	2	1	0	0	0	2	17
Total Approvals	**18**	**0**	**3**	**2**	**1**	**2**	**1**	**1**	**2**	**2**	**1**	**0**	**0**	**1**	**2**	**36**
Pending Approval	0	0	1	0	2	0	0	0	1	1	0	0	0	0	0	5
Approval Rate (%)	**46.2**	**0.0**	**37.5**	**40.0**	**20.0**	**50.0**	**12.5**	**10.0**	**28.6**	**66.7**	**50.0**	**0.0**	**0.0**	**33.3**	**100.0**	**32.1**
Immediate Rate (%)	33.3	0.0	25.0	0.0	0.0	50.0	12.5	0.0	0.0	0.0	0.0	0.0	0.0	33.3	0.0	17.0
Potential Rate (%)	46.2	0.0	50.0	40.0	60.0	50.0	12.5	10.0	42.9	100.0	50.0	0.0	0.0	33.3	100.0	36.6

D4 Spring 2021	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	v13	v14	Total
Attempts	**27**	**14**	**13**	**14**	**19**	**10**	**12**	**9**	**9**	**10**	**7**	**5**	**5**	**2**	**156**
Total Approvals	**3**	**0**	**2**	**0**	**4**	**3**	**0**	**2**	**4**	**0**	**0**	**0**	**1**	**0**	**19**
Pending Approval	2	1	3	3	1	2	2	2	0	2	2	1	0	1	22
Approval Rate (%)	**11.1**	**0.0**	**15.4**	**0.0**	**21.1**	**30.0**	**0.0**	**22.2**	**44.4**	**0.0**	**0.0**	**0.0**	**20.0**	**0.0**	**12.2**
Potential Rate (%)	18.5	7.1	38.5	21.4	26.3	50.0	16.7	44.4	44.4	20.0	28.6	20.0	20.0	50.0	26.3

Commentary. This problem exposes multiple kinds of misconceptions and difficulties, including:

- Continued difficulty with getting components of inclined vectors;
- Difficulty expressing horizontal and vertical vectors ("too easy it's hard");
- Lack of clarity to distinguish known vs. unknown forces;
- Discomfort using solvers.

*D3 was renumbered D4 for Spring 2021. A new direction "Do NOT solve the equations" was introduced to focus on writing equations, and qualitative estimation of the sign or magnitude of a force using the force polygon.

D4 Level Mastery Test: Moment Calculations*

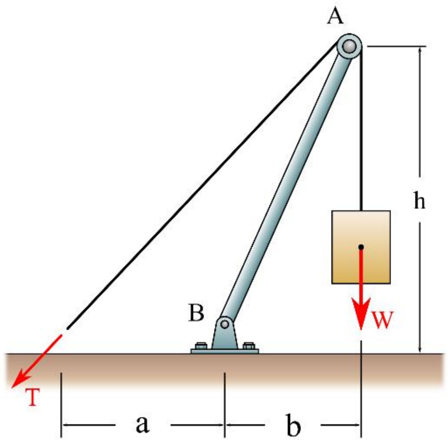
Problem Model.

In the system given, a = 14.1 m, b = 6.5 m, h = 41.7 m, and T = 51 kN.

1. Determine the moment of the force T about the point B in units of kN m. The value entered should be positive if it is counterclockwise; negative if it is clockwise.

2. If the force T is moved to point A, the value of the moment that it creates about point B

   a. increases
   b. remains the same
   c. decreases



Results.

D4 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	Total
Attempts	**39**	**31**	**11**	**6**	**6**	**11**	**15**	**13**	**1**	**5**	**5**	**5**	**148**
Immediate Approval	2	1	1	3	1	0	1	1	1	1	0	1	13
Conditional Approval	2	2	0	1	0	1	3	4	0	1	2	3	19
Total Approvals	**4**	**3**	**1**	**4**	**1**	**1**	**4**	**5**	**1**	**2**	**2**	**4**	**32**
Pending Approval	1	0	0	0	0	0	1	0	0	0	0	1	3
Approval Rate (%)	**10.3**	**9.7**	**9.1**	**66.7**	**16.7**	**9.1**	**26.7**	**38.5**	**100.0**	**40.0**	**40.0**	**80.0**	**21.6**
Immediate Rate (%)	5.1	3.2	9.1	50.0	16.7	0.0	6.7	7.7	100.0	20.0	0.0	20.0	8.8
Potential Rate (%)	12.8	9.7	9.1	66.7	16.7	9.1	33.3	38.5	100.0	40.0	40.0	100.0	23.6

D5 Spring 2021	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	v13	Total
Attempts	**19**	**21**	**11**	**14**	**9**	**16**	**14**	**11**	**8**	**4**	**3**	**3**	**2**	**135**
Total Approvals	**1**	**2**	**3**	**3**	**1**	**1**	**2**	**2**	**1**	**1**	**1**	**0**	**1**	**19**
Pending Approval	4	2	0	0	0	3	1	0	4	1	1	0	0	16
Approval Rate (%)	**5.3**	**9.5**	**27.3**	**21.4**	**11.1**	**6.3**	**14.3**	**18.2**	**12.5**	**25.0**	**33.3**	**0.0**	**50.0**	**14.1**
Potential Rate (%)	26.3	19.0	27.3	21.4	11.1	25.0	21.4	18.2	62.5	50.0	66.7	0.0	50.0	25.9

Commentary. Students display difficulty visualizing the perpendicular distance between a line and a point. Even after repeated requests and feedback, they do not instinctively sketch this or the line of action of the force. They are also uneasy using vector components to determine moments, and will oven multiply force components by distances that are parallel, and not perpendicular to, the component. They have further difficulty visualizing the direction of the moment.

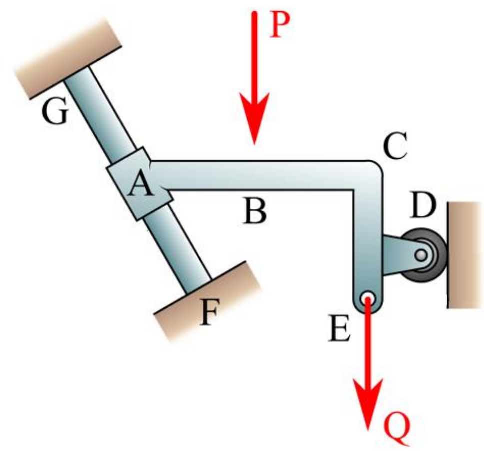
*D4 was renumbered to D5 for Spring 2021. No significant changes were made to the format.

D5 Level Mastery Test: Free Body Diagrams of Rigid Bodies*

Problem Model.

A light bracket *ABCDE* supports given loads *P* and *Q*, as shown. Making reasonable assumptions and identifications of the support types at *A* and *D*, draw consistent Free Body Diagrams of the following bodies:

(a) the bracket *ABCDE*, not including the wheel at *D*;
(b) the wheel at *D*;
(c) the bracket *ABCDE*, including the wheel at *D*;
(d) the rail *GF;*
(e) the entire system assembled, but separated from the support points.



Results.

D5 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	Total
Attempts	**34**	**30**	**11**	**4**	**7**	**8**	**7**	**7**	**4**	**1**	**1**	**1**	**115**
Immediate Approval	4	4	1	0	0	3	3	3	0	0	0	0	18
Conditional Approval	2	2	3	0	2	2	1	1	0	1	1	0	15
Total Approvals	**6**	**6**	**4**	**0**	**2**	**5**	**4**	**4**	**0**	**1**	**1**	**0**	**33**
Pending Approval	0	0	1	0	0	0	0	0	0	0	0	1	2
Approval Rate (%)	**17.6**	**20.0**	**36.4**	**0.0**	**28.6**	**62.5**	**57.1**	**57.1**	**0.0**	**100.0**	**100.0**	**0.0**	**28.7**
Immediate Rate (%)	11.8	13.3	9.1	0.0	0.0	37.5	42.9	42.9	0.0	0.0	0.0	0.0	15.7
Potential Rate (%)	17.6	20.0	45.5	0.0	28.6	62.5	57.1	57.1	0.0	100.0	100.0	100.0	30.4

D3 Spring 2021	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	v13	v14	v15	Total
Attempts	**45**	**44**	**20**	**26**	**18**	**22**	**5**	**20**	**13**	**9**	**9**	**7**	**7**	**6**	**2**	**253**
Total Approvals	**1**	**3**	**1**	**3**	**0**	**3**	**0**	**4**	**1**	**2**	**2**	**0**	**2**	**0**	**0**	**22**
Pending Approval	0	2	0	0	1	2	0	3	3	0	0	1	2	1	0	15
Approval Rate (%)	**2.2**	**6.8**	**5.0**	**11.5**	**0.0**	**13.6**	**0.0**	**20.0**	**7.7**	**22.2**	**22.2**	**0.0**	**28.6**	**0.0**	**0.0**	**8.7**
Potential Rate (%)	2.2	11.4	5.0	11.5	5.6	22.7	0.0	35.0	30.8	22.2	22.2	14.3	57.1	16.7	0.0	14.6

Commentary. Similar comments as for D2. In addition, students have difficulty identifying reaction types; they often confuse reactions as "causing" motion rather than preventing or resisting it. They also express various combinations of ignoring forces and torques that are present, and inserting superfluous ones that are not present. They also have difficulty using consistent labels for forces and torques that appear multiply, either as part of action reaction pairs, or as external reactions common to more than one subsystem.

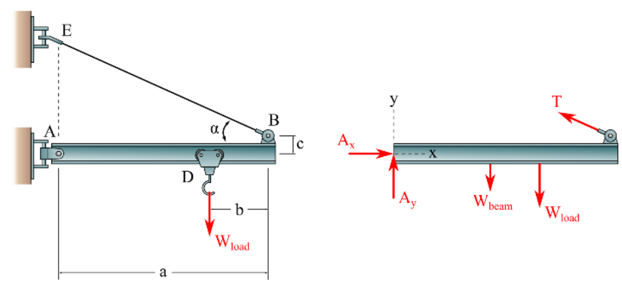
*The D5 test was renumbered to D3 for Spring 2021. No major changes were introduced, but students were required to explicitly identify action-reaction pairs.

D6 Level Mastery Test: Equilibrium of Rigid Bodies

Problem Model.

A beam is supported by a hinge at *A* and a cable at *B*. It carries a load at the end of the hook, W_{load} = 37 kN. The beam also has a weight W_{beam} = 2.49 kN, located at the midpoint of the beam. The geometric parameters are: *a* = 8.3 m, *b* = 2.8 m, c = 0.3 m, and α = 59º. A Free Body Diagram of the beam is provided at the right.

1. Using the given Free Body Diagram, write the Equations of Equilibrium that would allow you to solve for the unknown reactions, arranged such that the unknown variables are on the left, and given data are on the right.
2. After writing your Equations, solve the equations and enter the results into the answer boxes. Include the sign of each result, not just the magnitude. You may solve by hand or use the pre-programmed solver.
3. After determining the solution, draw the final force polygon of all forces, to approximate scale, showing that the resultant of the force vectors is zero.



Results.

D6 Fall 2020	**v1**	**v2**	**v3**	**v4**	**v5**	**v6**	**v7**	**v8**	**v9**	**v10**	**v11**	**v12**	**v13**	**Total**
Attempts	**34**	**27**	**4**	**4**	**15**	**17**	**3**	**8**	**12**	**5**	**7**	**5**	**2**	**143**
Immediate Approval	4	2	0	1	1	0	0	0	0	0	1	0	0	9
Conditional Approval	1	1	1	2	2	5	2	1	2	1	1	2	1	22
Total Approvals	**5**	**3**	**1**	**3**	**3**	**5**	**2**	**1**	**2**	**1**	**2**	**2**	**1**	**31**
Pending Approval	0	0	0	0	0	1	0	0	0	0	1	0	0	2
Approval Rate (%)	**14.7**	**11.1**	**25.0**	**75.0**	**20.0**	**29.4**	**66.7**	**12.5**	**16.7**	**20.0**	**28.6**	**40.0**	**50.0**	**21.7**
Immediate Rate (%)	11.8	7.4	0.0	25.0	6.7	0.0	0.0	0.0	0.0	0.0	14.3	0.0	0.0	6.3
Potential Rate (%)	14.7	11.1	25.0	75.0	20.0	35.3	66.7	12.5	16.7	20.0	42.9	40.0	50.0	23.1

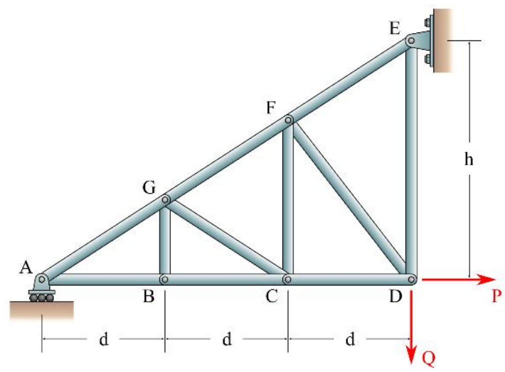
D6 Spring 2021	**v1**	**v2**	**v3**	**v4**	**v5**	**v6**	**v7**	**v8**	**v9**	**v10**	**v11**	**v12**	**Total**
Attempts	**21**	**8**	**7**	**4**	**18**	**6**	**8**	**4**	**5**	**9**	**10**	**2**	**102**
Total Approvals	**2**	**2**	**1**	**0**	**4**	**0**	**2**	**1**	**1**	**0**	**2**	**1**	**16**
Pending Approval	0	2	0	1	2	0	1	0	1	1	3	0	11
Approval Rate (%)	**9.5**	**25.0**	**14.3**	**0.0**	**22.2**	**0.0**	**25.0**	**25.0**	**20.0**	**0.0**	**20.0**	**50.0**	**15.7**
Potential Rate (%)	9.5	50.0	14.3	25.0	33.3	0.0	37.5	25.0	40.0	11.1	50.0	50.0	26.5

Commentary. Similar errors as for D3. In addition, there is continued difficulty calculating moments, particularly now when the forces have unknown values and are inclined. There is confusion between directly applied torque and the process of summing moments. There is also hesitation to recognize how to eliminate unknown forces by choosing equations strategically.

C1 Mastery Level Test: Trusses/Method of Joints and Zero Force Members

Problem Model.

<table>
<tr>
<td>Consider the following truss in which d = 4.3 <code>ft</code>, h = 10.3 <code>ft</code>, P = 159 <code>lbs</code>, and Q = 181 <code>lbs</code>.
1. Determine the reaction at A and the forces in members AB and AG. You may do this using as few equations as possible. You MUST include necessary Free Body Diagrams and Equations in your explanations, and draw a force polygon showing that all of the forces at joint A add to zero.

2. Identify all of the zero force members that can be identified by inspection.</td>
<td>
</td>
</tr>
</table>

Results.

C1 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	Total
Attempts	**32**	**15**	**2**	**9**	**6**	**7**	**4**	**2**	**2**	**2**	**3**	**84**
Immediate Approval	3	0	0	1	0	0	0	0	0	0	0	4
Conditional Approval	5	1	2	6	2	2	1	1	1	0	0	21
Total Approvals	**8**	**1**	**2**	**7**	**2**	**2**	**1**	**1**	**1**	**0**	**0**	**25**
Pending Approval	0	0	0	0	0	2	0	0	0	0	0	2
Approval Rate (%)	**25.0**	**6.7**	**100.0**	**77.8**	**33.3**	**28.6**	**25.0**	**50.0**	**50.0**	**0.0**	**0.0**	**29.8**
Immediate Rate (%)	9.4	0.0	0.0	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.8
Potential Rate (%)	25.0	6.7	100.0	77.8	33.3	57.1	25.0	50.0	50.0	0.0	0.0	32.1

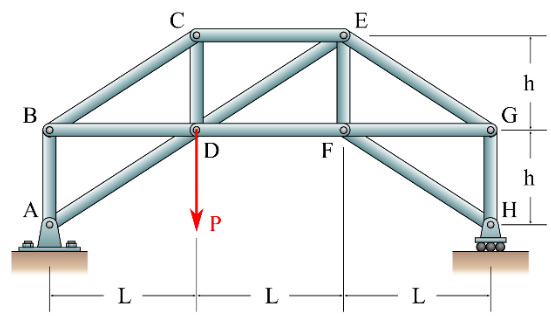
C1 Spring 2021	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	Total
Attempts	**13**	**5**	**7**	**9**	**8**	**6**	**10**	**9**	**2**	**3**	**72**
Total Approvals	**1**	**1**	**4**	**0**	**1**	**0**	**1**	**1**	**0**	**0**	**9**
Pending Approval	1	1	0	1	0	0	3	5	0	2	13
Approval Rate (%)	**7.7**	**20.0**	**57.1**	**0.0**	**12.5**	**0.0**	**10.0**	**11.1**	**0.0**	**0.0**	**12.5**
Potential Rate (%)	15.4	40.0	57.1	11.1	12.5	0.0	40.0	66.7	0.0	66.7	**30.6**

Commentary. Unlike in D3 and D6, in which Free Body Diagrams were provided, here students are responsible for drawing their own FBD's. A common error is confusion between drawing the joint FBD vs. the FBD of the entire system; both are often required for the C1 test. Frequently, forces applied externally to the truss at other points are incorrectly transferred to the joint FBD. Some students attempt to draw a torque at the joint and/or apply a moment equation. Problems with obliquely oriented reactions also cause difficulties.

C2 Mastery Test Practice

Problem Model.

In the following truss, L = 8.5 `ft`, h = 5.4 `ft`, and P = 3.9 `kips`. Determine the forces in members *CE*, *DE*, and *DF*. Show all work.



Results.

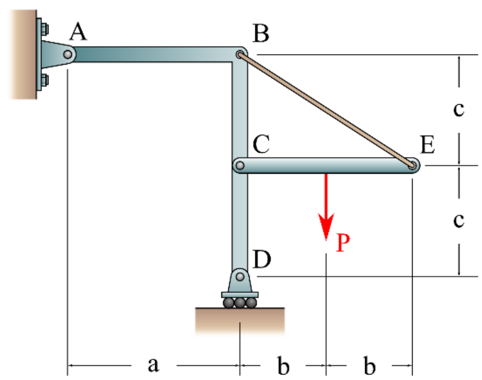
C2 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	Total
Attempts	**31**	**19**	**6**	**6**	**6**	**9**	**7**	**4**	**4**	**3**	**1**	**96**
Immediate Approval	2	2	0	1	0	1	3	0	1	0	0	10
Conditional Approval	4	1	0	1	3	1	1	2	1	1	0	15
Total Approvals	**6**	**3**	**0**	**2**	**3**	**2**	**4**	**2**	**2**	**1**	**0**	**25**
Pending Approval	1	0	0	0	0	0	0	0	0	1	0	2
Approval Rate (%)	**19.4**	**15.8**	**0.0**	**33.3**	**50.0**	**22.2**	**57.1**	**50.0**	**50.0**	**33.3**	**0.0**	**26.0**
Immediate Rate (%)	6.5	10.5	0.0	16.7	0.0	11.1	42.9	0.0	25.0	0.0	0.0	10.4
Potential Rate (%)	22.6	15.8	0.0	33.3	50.0	22.2	57.1	50.0	50.0	66.7	0.0	28.1

C2 Spring 2021	v1	v2	v3	v4	v5	v6	v7	v8	v9	Total
Attempts	**8**	**4**	**9**	**7**	**9**	**11**	**7**	**4**	**1**	**60**
Total Approvals	**2**	**0**	**0**	**4**	**0**	**1**	**1**	**2**	**0**	**10**
Pending Approval	1	0	1	2	1	1	1	1	0	8
Approval Rate (%)	**25.0**	**0.0**	**0.0**	**57.1**	**0.0**	**9.1**	**14.3**	**50.0**	**0.0**	**16.7**
Potential Rate (%)	37.5	0.0	11.1	85.7	11.1	18.2	28.6	75.0	0.0	30.0

Commentary. Similar errors from C1 persist, such as the distinction between the FBD's of the section vs. the entire truss. Some students have difficulty identifying the appropriate section. Some students omit the necessary moment equation, although there is some progress since D6 to recognize the strategic points around which to calculate moments. In cases when a section can be cantilevered, students often do not recognize that, and by default, solve for the reactions on the entire truss first.

C3 Mastery Level Practice

Problem Model.

Consider the following frame in which a = 6.2 `ft`, b = 4.8 `ft`, c = 4 `ft`, and P = 301 `lbs`. Determine the internal forces and/or torques acting on member *ABCD* at points *A*, *B*, *C*, and *D*. Justify your work by showing necessary Free Body Diagrams and corresponding equations. You may use any combination of equations from the entire structure or subsystems, and you may use as few equations as possible.	

Results.

C3 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	Total
Attempts	**29**	**16**	**5**	**10**	**12**	**7**	**8**	**7**	**6**	**3**	**103**
Immediate Approval	2	0	1	0	2	0	1	0	0	1	7
Conditional Approval	2	1	1	0	3	1	0	2	2	0	12
Total Approvals	**4**	**1**	**2**	**0**	**5**	**1**	**1**	**2**	**2**	**1**	**19**
Pending Approval	1	0	0	0	0	0	0	2	1	0	4
Approval Rate (%)	**13.8**	**6.3**	**40.0**	**0.0**	**41.7**	**14.3**	**12.5**	**28.6**	**33.3**	**33.3**	**18.4**
Immediate Rate (%)	6.9	0.0	20.0	0.0	16.7	0.0	12.5	0.0	0.0	33.3	6.8
Potential Rate (%)	17.2	6.3	40.0	0.0	41.7	14.3	12.5	57.1	50.0	33.3	22.3

C3 S2	v1	v2	v3	v4	v5	v6	v7	v8	v9	Total
Attempts	**5**	**5**	**9**	**5**	**8**	**12**	**7**	**6**	**1**	**58**
Total Approvals	**0**	**0**	**0**	**0**	**0**	**3**	**2**	**4**	**0**	**9**
Pending Approval	2	0	0	0	0	0	0	0	1	3
Approval Rate (%)	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**25.0**	**28.6**	**66.7**	**0.0**	**15.5**
Potential Rate (%)	40.0	0.0	0.0	0.0	0.0	25.0	28.6	66.7	100.0	19.0

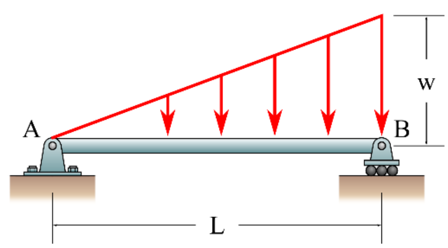
Commentary. In C3, the number of equations grows, and students might have difficulty keeping track of the same variables that are common to multiple subsystems. Some students want to continue using "truss mentality" and consider forces parallel to beams and columns. Yet ironically, they equally have difficulty identifying truly 2-force members in a frame, and typically choose to draw the member as if it were a general 2D body. Some students who have facility to draw the proper FBD's get stuck when it is necessary to use more than one body to solve the equations. Sign conventions issues creep up again, particular when solved values for reactions are negative.

B1 Mastery Level Practice

Problem Model.

A beam has a distributed load as shown, with w = 31 `lbs/ft` and L = 12 `ft`. Determine the internal shear and moment reactions at the point x = 1.2 `ft`.

1. For shear force, report the value as positive if it acts downward on the left section of the beam/upward on the right section; report the value as negative if it is opposite of this.
2. For moment, report the value as positive if it acts counterclockwise on the left section of the beam/clockwise on the right section of the beam; report the value as negative if it is opposite of this.



Results.

B1 Fall 2020	v1	v2	v3	v4	v5	v6	v7	v8	v9	Total
Attempts	**20**	**5**	**5**	**1**	**6**	**4**	**5**	**7**	**4**	**57**
Immediate Approval	2	0	1	0	0	1	0	1	0	5
Conditional Approval	4	1	2	0	1	0	1	1	3	13
Total Approvals	**6**	**1**	**3**	**0**	**1**	**1**	**1**	**2**	**3**	**18**
Pending Approval	0	0	0	0	0	0	0	0	0	0
Approval Rate (%)	**30.0**	**20.0**	**60.0**	**0.0**	**16.7**	**25.0**	**20.0**	**28.6**	**75.0**	**31.6**
Immediate Rate (%)	10.0	0.0	20.0	0.0	0.0	25.0	0.0	14.3	0.0	8.8
Potential Rate (%)	30.0	20.0	60.0	0.0	16.7	25.0	20.0	28.6	75.0	31.6

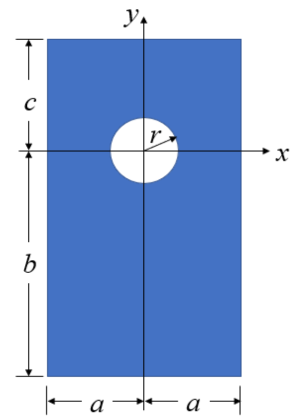
B1 Spring 2021	v1	v2	v3	v4	v5	v6	v7	Total
Attempts	**5**	**7**	**4**	**4**	**2**	**2**	**2**	**26**
Total Approvals	**1**	**1**	**0**	**1**	**0**	**1**	**0**	**4**
Pending Approval	0	2	1	0	0	0	2	5
Approval Rate (%)	**20.0**	**14.3**	**0.0**	**25.0**	**0.0**	**50.0**	**0.0**	**15.4**
Potential Rate (%)	20.0	42.9	25.0	25.0	0.0	50.0	100.0	34.6

Commentary. This problem presents difficulties both “at level”, such as not understanding how to make intermediate cuts or understand the (new) idea of centroid. But there are also “below level” difficulties that require equal time to address; even when students do see the proper cut, and they see how the cut divides the distributed load, they are not sufficiently fluent with the use of similar triangles and proportionality equations that are required to calculate the intermediate intensities of the distributed loads. The relatively high passing rate (compared with C3) is probably a sign that by this point in the course, the students taking this test are fewer in number, self-selected as the ones that feel in position to complete this level.

B2/B3 Mastery Level: Area Centroid and Area Moment of Inertia

Problem Model.

Consider the following region, where a = 10.5 `in`, b = 18 `in`, c = 10.3 `in`, and r = 5.8 `in`. Determine the location of the centroid relative to the given x and y axes, and also the moment of inertia about the horizontal axis that passes through the centroid.



Results.

B2/B3 Fall 2020	v1	v2	v3	v4	v5	v6	v7	Total
Attempts	**19**	**4**	**3**	**4**	**7**	**3**	**1**	**41**
Immediate Approval	4	0	0	1	0	0	0	5
Conditional Approval	3	0	2	0	5	2	0	12
Total Approvals	**7**	**0**	**2**	**1**	**5**	**2**	**0**	**17**
Pending Approval	0	0	0	0	0	0	1	1
Approval Rate (%)	**36.8**	**0.0**	**66.7**	**25.0**	**71.4**	**66.7**	**0.0**	**41.5**
Immediate Rate (%)	21.1	0.0	0.0	25.0	0.0	0.0	0.0	12.2
Potential Rate (%)	36.8	0.0	66.7	25.0	71.4	66.7	100.0	43.9

B2/B3 Spring 2021	v1	v2	v3	v4	v5	Total
Attempts	**6**	**3**	**7**	**5**	**2**	**23**
Total Approvals	**2**	**0**	**2**	**1**	**0**	**5**
Pending Approval	0	1	0	0	0	1
Approval Rate (%)	**33.3**	**0.0**	**28.6**	**20.0**	**0.0**	**21.7**
Potential Rate (%)	33.3	33.3	28.6	20.0	0.0	26.1

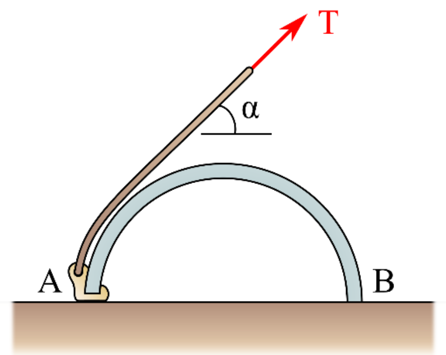
Commentary. Students generally grasp the centroid equations, but have the common error of not measuring all of the positions from a common reference, e.g., the position of the centroid of a triangle is "always" at 1/3 of the length of the side. There is somewhat more difficulty with the moment of inertia, wherein students will often ignore the "Ad^2" term. A clear visual approach showing the distinction between the centroidal axes of the sub-shapes vs. the aggregate centroidal axis appears to be helpful.

A1 Mastery Level: Friction

Problem Model.

A cable pulls on a semi-circular pipe section that has weight $W = 164$ `lbs`. If the coefficient of static friction between the pipe and the floor is $\mu = 0.15$, and the angle $\alpha = 59^o$, determine the tension T required to initiate movement of the pipe and whether the pipe will slide or tip.

Hint: Assume impending motion in each case, and compare the results of each case to determine which one occurs first.



Results.

A1 Fall 2020	v1	v2	v3	v4	v5	v6	Total
Attempts	**14**	**4**	**2**	**3**	**5**	**2**	**30**
Immediate Approval	2	0	0	0	1	0	3
Conditional Approval	3	2	1	0	2	0	8
Total Approvals	**5**	**2**	**1**	**0**	**3**	**0**	**11**
Pending Approval	1	0	0	0	0	0	1
Approval Rate (%)	**35.7**	**50.0**	**50.0**	**0.0**	**60.0**	**0.0**	**36.7**
Immediate Rate (%)	14.3	0.0	0.0	0.0	20.0	0.0	10.0
Potential Rate (%)	42.9	50.0	50.0	0.0	60.0	0.0	40.0

A1 Spring 2021	v1	v2	v3	Total
Attempts	2	8	7	**17**
Total Approvals	**0**	**1**	**1**	**2**
Pending Approval	0	1	1	2
Approval Rate (%)	0.0	12.5	14.3	11.8
Potential Rate (%)	0.0	25.0	28.6	23.5

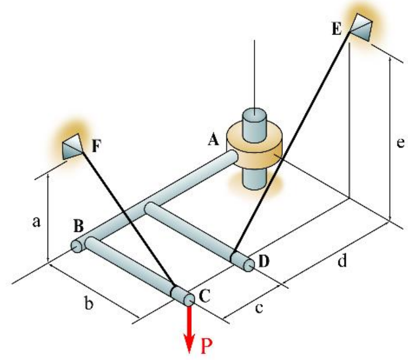
Commentary. The principal issue here is that students are unable to navigate the logical structure of friction problems, in which it is typically required to make an assumption of a case or mode, solve the system under that assumption, check the assumption, and then re-solve the problem if necessary. Compounding the problem is that some problems present the dichotomy of static vs. dynamic friction, whereas others present the assumption of impending motion. Some students who “know the mechanics” under a well posed set of assumptions are unable to overcome the challenge of thinking “algorithmically” on a set of logical tests.

A2/A3 Mastery Level Test: 3D Vector Operations and Rigid Body Equilibrium

Problem Model.

A rigid element is supported by a smooth, round pole at *A* and by cables T_{CF} and T_{DE}, as shown. A vertical force P = 191 N is applied at *C*, as shown. The geometrical dimensions are a = 37 cm, b = 59 cm, c = 29 cm, d = 50 cm, and e = 54 cm. Note that the figure may not be drawn to scale with your given values.

1. Draw the appropriate Free Body Diagram required that will allow you to solve for the forces in the cables.
2. Write a set of equations that will allow you to solve for the forces in the cables. NOTE: you can do this using as few equations as possible, without necessarily writing a complete set of equilibrium equations.
3. After writing your equations, solve them by any method.



Results.

A2/A3 Fall 2020	v1	v2	v3	Total
Attempts	**12**	**7**	**9**	**28**
Immediate Approval	4	0	1	5
Conditional Approval	0	0	6	6
Total Approvals	**4**	**0**	**7**	**11**
Pending Approval	12	7	9	28
Approval Rate (%)	**33.3**	**0.0**	**77.8**	**39.3**
Immediate Rate (%)	33.3	0.0	11.1	17.9
Potential Rate (%)	33.3	0.0	88.9	42.9

A2/A3 Spring 2021	v1	v2	v3	Total
Attempts	**2**	**4**	**1**	**7**
Total Approvals	**0**	**1**	**1**	**2**
Pending Approval	0	0	0	0
Approval Rate (%)	**0.0**	**25.0**	**100.0**	**28.6**
Potential Rate (%)	0.0	25.0	100.0	28.6

Commentary. This level had relatively few attempts by virtue of both the limitation in the calendar at the time the topic is delivered, and also accounting for even fewer students who feel in position to make the attempt. However, a surprising number of “B” students made a strong effort at the end and mastered the topic. Difficulties included understanding how to take moments around axes, particularly as part of a strategy to eliminate other unknown reactions. The different rules that eliminate forces (going through or parallel to an axes) vs. torques (perpendicular to an axes) required repeated conversations to master.